Quiz 7 Solutions

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1. For what values of k is the matrix $A = \begin{pmatrix} 0 & 1 & k \\ 3 & 2k & 5 \\ 9 & 7 & 5 \end{pmatrix}$ invertible?

Solution: Expanding the determinant out along the first row, we have $\det(A) = 30 + k(21 - 18k) = 30 + 21k - 18k^2$. A is invertible if and only if $\det(A) \neq 0$. We have $30 - 21k + 18k^2 = -3(k-2)(6k+5)$, so A is invertible if and only if $k \neq 2, -5/6$.

2. Let A be an $n \times n$ matrix with $\det(A) = 3$. What are the possible values of the following:

(a) $\det(A^t)$

(b) $\det(A^t A)$

(c) $\det(Q)$ and $\det(R)$ where $A = QR$ is a QR factorization.

Solution:

(a) $\det(A^t) = \det(A) = 3$.

(b) $\det(A^t A) = \det(A)^2 = 9$.

(c) Since Q is orthogonal, $\det(Q) = \pm 1$, and we have $\det(A) = \det(Q)\det(R)$. By the textbook's construction of the QR-factorization, $\det(R)$ is necessarily positive, so we find $\det(R) = 3$.

3. Find two matrices A and B such that $\det(A) = \det(B)$ but A and B are not similar.

Solution: Let $A, B \in \mathbb{R}^{3\times 3}$ where A is the matrix of the orthogonal projection onto the line in the direction of the vector $(1,1,1)$ and B be the matrix of the orthogonal projection onto the xy-plane. Then A and B are both not invertible, so $\det(A) = \det(B) = 0$. However, $\operatorname{rank}(A) = 1$ while $\operatorname{rank}(B) = 2$, so A and B are not similar.