## Quiz 6 Solutions Tim Smits

1. Find the QR factorization of  $A = \begin{pmatrix} 1 & 1 \\ 1 & 9 \\ 1 & -5 \\ 1 & 3 \end{pmatrix}$ 

**Solution:** Let the columns of A be  $v_1$  and  $v_2$ . Set  $u_1 = v_1/||v_1|| = v_1/2 = (1/2, 1/2, 1/2, 1/2)$ . We then have  $v_2^{\perp} = v_2 - (u_1 \cdot v_2)u_1 = v_2 - 4u_1 = (-1, 7, -7, 1)$ . Take  $u_2 = v_2^{\perp}/||v_2^{\perp}|| = v_2^{\perp}/10 = (-1/10, 7/10, -7/10, 1/10)$ . We then have  $v_1 = 2u_1$  and  $v_2 = 4u_1 + 10u_2$ , so  $Q = \begin{pmatrix} 1/2 & -1/10 \\ 1/2 & 7/10 \\ 1/2 & 7/10 \\ 1/2 & 1/10 \end{pmatrix}$  and  $R = \begin{pmatrix} 2 & 4 \\ 0 & 10 \end{pmatrix}$ .

**2.** Find the best fit line  $f(t) = c_0 + c_1 t$  passing through (1, 10), (9, 0), (-5, 0), (3, 0).

**Solution:** Let  $b = \begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ . We need to find the least squares solution to Ax = b where A is the matrix from problem 1. This is equivalent to solving  $A^tAx = A^tb$ , and since A has rank 2,  $A^tA$  is invertible, so  $x = (A^tA)^{-1}A^tb$ . Plugging in says  $x = \begin{pmatrix} 4 & 8 \\ 8 & 116 \end{pmatrix}^{-1} \begin{pmatrix} 10 \\ 10 \end{pmatrix} = \begin{pmatrix} 27/10 \\ -1/10 \end{pmatrix}$ .