Quiz 5 Solutions

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1. Let V be the subspace spanned by the vectors $\vec{v}_1 = (1,1,1,1)$, $\vec{v}_2 = (1,1,-1,-1)$, $\vec{v}_3 = (1,-1,-1,1)$ in $\mathbb{R}^4$. Compute $\text{Proj}_V(\vec{e}_1)$.

Solution: Normalize $\vec{v}_1, \vec{v}_2, \vec{v}_3$ to get unit vectors $\vec{u}_1 = (1/2, 1/2, 1/2, 1/2)$, $\vec{u}_2 = (1/2, 1/2, -1/2, -1/2)$ and $\vec{u}_3 = (1/2, 1/2, -1/2, -1/2)$. Then $\text{Proj}_V(\vec{e}_1) = (\vec{e}_1 \cdot \vec{u}_1)\vec{u}_1 + (\vec{e}_1 \cdot \vec{u}_2)\vec{u}_2 + (\vec{e}_1 \cdot \vec{u}_3)\vec{u}_3 = \frac{1}{2}\vec{u}_1 + \frac{1}{2}\vec{u}_2 + \frac{1}{2}\vec{u}_3 = (3/4, 1/4, -1/4, 1/4)$.

2. Find $\vec{u}_4$ such that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ is an orthonormal basis of $\mathbb{R}^4$, where $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are the normalizations of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

Solution: Any unit vector orthogonal to $\vec{u}_1, \vec{u}_2$, and $\vec{u}_3$ will work. We have $\vec{e}_1^{\perp} = \vec{e}_1 - \text{Proj}_V(\vec{e}_1) \in V^{\perp}$, so we can take $\vec{u}_4 = \vec{e}_1^{\perp}/\|\vec{e}_1^{\perp}\|$. We see that $\vec{e}_1^{\perp} = (1,0,0,0) - (3/4, 1/4, -1/4, 1/4) = (1/4, -1/4, 1/4, -1/4)$, so normalizing gives $\vec{u}_4 = (1/2, -1/2, 1/2, -1/2)$.