Quiz 3 Solutions

Tim Smits

1. Write down the definition of linear dependence and use it to show that the set $S = \{(1,1,1),(1,0,0),(0,1,0),(0,0,1)\}$ is linearly dependent.

Solution: A set S is linearly dependent if there are vectors $\vec{v}_1, \ldots, \vec{v}_n \in S$ and constants $c_1, \ldots, c_n$ not all 0 such that $c_1\vec{v}_1 + \ldots + c_n\vec{v}_n = \vec{0}$. In our case, notice that $(1,1,1) - (1,0,0) - (0,1,0) - (0,0,1) = (0,0,0)$, which says S is linearly dependent.

2. True or false?

(a) Invertible matrices have linearly independent columns.

(b) $\text{im}(T)$ is spanned by the rows of the matrix A where $T(\vec{x}) = A\vec{x}$.

(c) Invertible matrices are always invertible linear transformations, i.e. if $A \in \mathbb{R}^{n\times n}$ is invertible, then $T(\vec{x}) = A\vec{x}$ defines an invertible map.

(d) If $T : \mathbb{R}^2 \to \mathbb{R}^4$ is a linear transformation, then T cannot be injective.

Solution:

(a) True; see problem 3.2.45.

(b) False; $\text{im}(T)$ is spanned by the columns of A.

(c) True; If $T(\vec{x}) = \vec{0}$, this says $A\vec{x} = \vec{0}$. Since A is invertible, this says $\vec{x} = \vec{0}$, so $\ker(T) = \{\vec{0}\}$ says that T is injective. By rank-nullity, $\dim(\text{im}(T)) = n$, so that $\text{im}(T) = \mathbb{R}^n$ says T is surjective.

(d) False; The map $T((x,y)) = (x,y,0,0)$ is linear, and also injective.