# Quiz 2 Solutions

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**1.** Let $f(x,y) = \begin{pmatrix} 2x+y \\ x-y \end{pmatrix}$, and $g(x,y) = \begin{pmatrix} x+y \\ -y \end{pmatrix}$. Define $h(x,y) = g \circ f(x,y)$. Find the matrix $A$ such that $h(x,y) = A \begin{bmatrix} x \\ y \end{bmatrix}$.

**Solution:** Let $B$ be the matrix of $g$ and let $C$ be the matrix of $f$. Since composition of linear transformations corresponds to multiplication of matrices, we have see that $A = BC$. To compute $B$ and $C$, we just need to compute what $f$ and $g$ do to the vectors $(1,0)$ and $(0,1)$. We see that $g(1,0) = (1,0)$, $g(0,1) = (1,-1)$, $f(1,0) = (2,1)$, and $f(0,1) = (1,-1)$. This says $B = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$, so that $A = \begin{pmatrix} 3 & 0 \\ -1 & 1 \end{pmatrix}$.

**2.** Let $\vec{n} = (1,2,3)$. Let $T$ be the projection onto the line $L = \{t\vec{n} : t \in \mathbb{R}\}$ and $S$ be the reflection about the plane $\{\vec{x} : \vec{x} \cdot \vec{n} = 0\}$. Are $T$ and $S$ invertible? Why or why not?

**Solution:** $T$ is not invertible, because it's not surjective: $T$ maps $\mathbb{R}^3$ onto the line $L$, which in particular, is not all of $\mathbb{R}^2$. $S$ is invertible, because reflecting a vector twice sends it back to the original vector. That is, $S$ is its own inverse.