Quiz 1 Solutions Tim Smits

1. Sketch the eight possible ways three planes can be positioned relative to each other.

Solution: See here.

2. Solve the following systems of equations:

(a)
$$\begin{cases} 2x + 3y = 2\\ 4x + 6y = 4 \end{cases}$$

(b)
$$\begin{cases} x + 2y + 3z = 1\\ 2x + 4y + 7z = 2\\ 3x + 7y + 11z = 8 \end{cases}$$

You may use any technique you want. If the system has no solution, explain why. If it has infinitely many solutions, describe the solution set (e.g. by parameterization).

Solution:

(a) The first equation is the same as the second equation (up to scalar multiple), so the solution to the system is determined by the equation 2x + 3y = 2. This says x = 1 - 3y/2, so setting y = t as a free parameter says solutions are given by (x, y) = (1 - 3t/2, t) for $t \in \mathbb{R}$.

(b) Write the corresponding augmented system $\begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 2 & 4 & 7 & | & 2 \\ 3 & 7 & 11 & | & 8 \end{pmatrix}$. Performing elimination: $\begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 2 & 4 & 7 & | & 2 \\ 3 & 7 & 11 & | & 8 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 1 \\ 0 & 0 & 1 & | & 0 \\ 0 & 1 & 2 & | & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & | & -9 \\ 0 & 0 & 1 & | & 0 \\ 0 & 1 & 0 & | & 5 \end{pmatrix}$. This says (x, y, z) = (-9, 5, 0) is the unique solution.