Improper Integral I

The Riemann Integral Cannot Inkegrate over "infinite" regions: · Lahnite Regions. (I)

· Fanchois that "blow up" on the Internel (II)

Goal: extend the integral to handle these Cases.

Type I: Infinite Region. $(-\infty,\infty)$ (-10, a], [a, 10) and

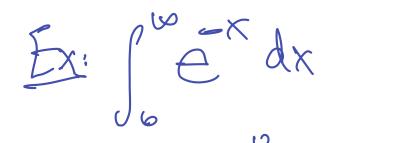
Start with [a,vo): Enkyrate aus larger and larger firste inkovels and defie as a limiting process. $\int_{a}^{w} f(z) dx \stackrel{def}{=} \lim_{R \to \infty} \int_{a}^{K} f(z) dx$ flada def lim flada -vo R-rvo R $\int f(x) dx = \int f(x) dx + \int f(x) dx$

for any ce (-co, vo). $\lim_{R_1 \to \infty} \int_{R_1}^{C} f(x) dx + \lim_{R_2 \to \infty} \int_{C}^{R_2} f(x) dx$ Note: this is not lim fex dx. R-700 fex dx. This "Symmetric" process is called the Cauchy Principal Value. Type II: Discontinuity. Very Similar i dea. Sippose that flat has ar infinite discontinuity at c $Ce(a_1b)$

 $\int_{\alpha}^{b} f(z) dz = \lim_{R \to C} \int_{\alpha}^{R_{1}} \int_{\alpha}^{R_{2}} \int_{\alpha}^{R_{2}$

Discontinuity at endpoint is handled similarly to above.

If the Senit is defined, we say the Impropri integrel converges and assign it the value of the limit. Otherwise, it diverges.





$$= \lim_{R \to \infty} \int_{0}^{R} e^{-x} dx = e^{-x} \int_{0}^{1} e^{-x} dx$$

$$= \lim_{R \to \infty} 1 - e^{-R} = 1$$

$$\frac{1}{2} \int_{0}^{\infty} \frac{1}{1 - e^{-R}} = 1$$

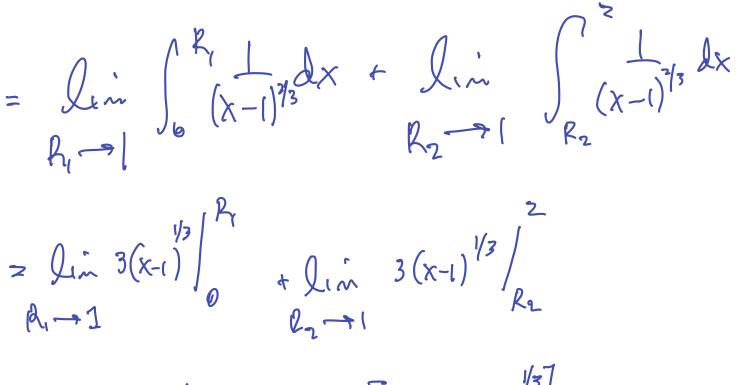
$$\frac{1}{2} \int_{0}^{\infty} \frac{1}{1 - e^{-R}} \int_{0}^{\infty} \frac{1$$

$$+ \lim_{R_2 \to \infty} \int_{0}^{R_2} \frac{1}{\mathrm{tr}(\mathrm{tr}x^2)} dx$$

$$= \lim_{R_1 \to \infty} \frac{1}{\mathrm{tr}} \frac{1}{\mathrm{tr}^{-1}(x)} \Big|_{R_1}^{0} + \lim_{R_2 \to \infty} \frac{1}{\mathrm{tr}} \frac{1}{\mathrm{tr}^{-1}(x)} \Big|_{0}^{R_2}$$

 $= \frac{1}{2} + \frac{1}{2} = 1.$

 $E_{X:} \int_{6}^{2} \frac{1}{(X-1)^{2}} dX = \int_{6}^{1} \frac{1}{(X-1)^{2}} dx + \int_{1}^{2} \frac{1}{(X-1)^{2}} dx$



 $= \lim_{R_{1} \to 1} \left[3(R_{1}-1)^{1/3} + 3 \right] + \lim_{R_{2} \to 1} \left[3 - 3(R_{2}-1)^{1/3} \right]$ = $(Q_{1}-1)^{1/3}$