Integration Strategies

- Substitute away anything you don't like. Algebraically changing the integral often makes it easier to see how to proceed!

- Check if common techniques apply:
  - trig sub
  - partial fractions
  - trig integrals

- Try IBP if nothing else works!

- IBP works well when there are obvious products of functions. Helpful w/ logs, inverse trig functions, etc

Examples

- $$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$u = \sqrt{x}$

$dx = 2u\,du$

$$\int 2e^{u}\,du$$

$$= 2e^{u} + C$$

$$= 2e^{\sqrt{x}} + C.$$

- $$\int \tan^{-1} x\,dx$$

$u = \tan^{-1}(x)$ $\qquad du = 1/1+x^2$

$dv = 1$ $\qquad v = x$

$$x\tan^{-1}(x) - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C.$$

$$\int \frac{x^3 - x^2 + 1}{(x^2+2x+1)(x^2+2x+2)}\,dx$$

Quite complicated!

$$\frac{x^3 - x^2 + 1}{(x+1)^2(x^2+2x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{(x+1)}{x^2+2x+2}$$

One lengthy computation later:

$$\frac{5}{x+1} - \frac{1}{(x+1)^2} - \frac{4x-7}{x^2+2x+2}$$

Easy

$$\int \frac{-4x-7}{(x+1)^2+1}\,dx \qquad u = x+1 \qquad x = u-1$$

$$\int \frac{-4u-11}{u^2+1}\,du$$

$$= -2\ln|u^2+1| - 11\tan^{-1}(u)$$

$$-5\ln|x+1| + \frac{1}{x+1} - 2\ln|x^2+2x+2|$$

$$-11\tan^{-1}(x^2+2x+2) + C.$$

- $$\int \sin^2(x)\tan(x)\,dx$$

$$\int \frac{\sin^3(x)}{\cos(x)}\,dx$$

$$= \int \frac{(1-\cos^2 x)\sin(x)}{\cos(x)}\, dx$$

$$\int \frac{u^2-1}{u}\, du$$

$$= \frac{1}{2}u^2 - \ln|u| + C$$

$$= \frac{1}{2}\cos^2(x) - \ln|\cos(x)| + C.$$

$$\int \frac{x}{(1-x^2)^4}\, dx$$

$$x = \sin\theta$$

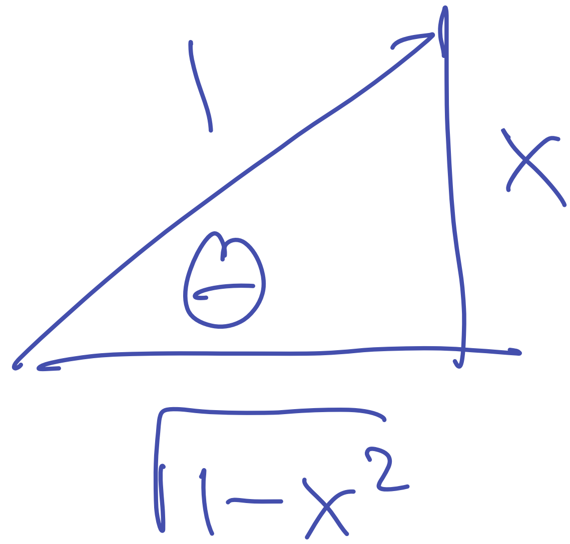
$$dx = \cos\theta \, d\theta$$

$$\int \frac{\sin\theta}{\cos^8\theta} \cdot \cos\theta \, d\theta$$

$$\int \frac{\sin\theta}{\cos^7\theta} \, d\theta \qquad u = \cos\theta$$

$$\int \frac{-du}{u^7} = \frac{1}{6u^6} + C$$

$$= \frac{1}{6\cos^6\theta} + C$$



$$\frac{1}{6(1-x^2)^3} + C$$