# Trig Sub

Exploit substitution to make trig identities appear.

| Expression | Substitute |
|---|---|
| $u^2 + a^2$ | $u = a\tan\theta$ |
| $a^2 - u^2$ | $u = a\sin\theta$ |
| $u^2 - a^2$ | $u = a\sec\theta$ |

Trig sub works quite well if you have radical expressions

e.g. $\sqrt{1+x^2}$ or whatnot.

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Ex: $\int \sqrt{1-x^2}\, dx$

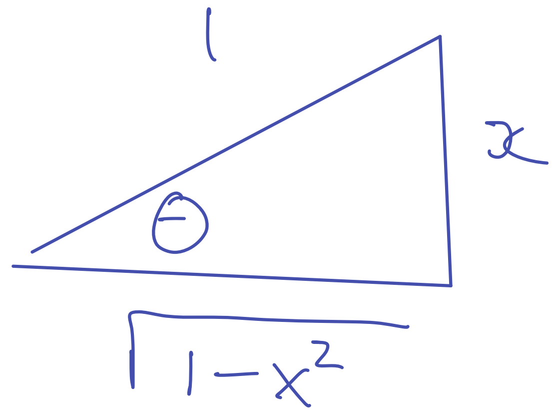
$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$$\int \cos^2\theta \, d\theta = \int \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta + C$$




$\sin\theta = x \qquad \theta = \sin^{-1}(x)$

$\cos\theta = \sqrt{1-x^2}$

$$= \frac{1}{2}\sin^{-1}(x) + \frac{1}{2}x\sqrt{1-x^2} + C.$$

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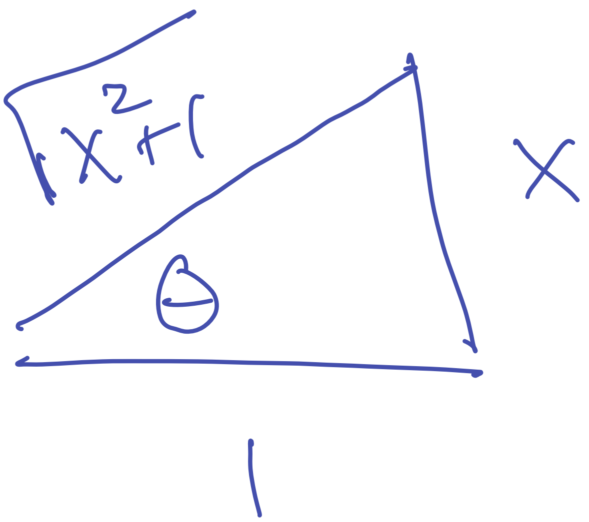
Ex:
$$\int \frac{1}{(1+x^2)^2}\,dx$$

$x = \tan\theta \qquad dx = \sec^2\theta\, d\theta$

$$\int \frac{1}{\sec^4\theta}\sec^2\theta\, d\theta$$

$$= \int \cos^2\theta\, d\theta = \frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta + C$$




$$\sin\theta = \frac{x}{\sqrt{x^2+1}}$$

$$\cos\theta = \frac{1}{\sqrt{x^2+1}}$$

$$= \tan^{-1}(x) + \frac{1}{2}\frac{x}{x^2+1} + C.$$

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<u>Ex:</u>

$$\int \frac{x^2}{\sqrt{x^2-1}}\,dx \qquad x = \sec\theta$$

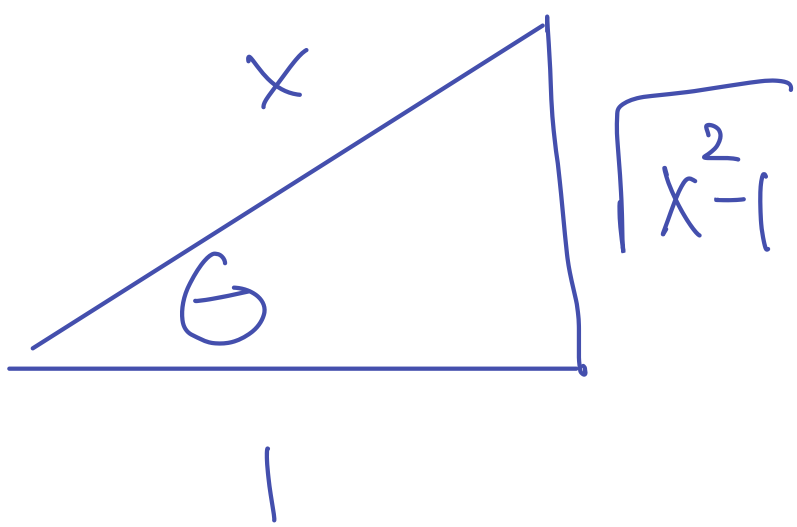
$$dx = \sec\theta\tan\theta\,d\theta$$

$$\int \frac{\sec^2\theta}{\tan\theta}\cdot \sec\theta\tan\theta\,d\theta$$

$$= \int \sec^3\theta\,d\theta$$

wed. lecture

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$




$\sec\theta = x$

$\tan\theta = \sqrt{x^2-1}$

$$= \frac{1}{2}x\sqrt{x^2-1} + \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C.$$

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General quadratic expressions:

$ax^2+bx+c$ can always

Complete the square to

make it look like one of

the above forms that trig

Sub will work on.

Ex: $\displaystyle\int \sqrt{x^2-4x+7}\,dx$

$$x^2-4x+7 = (x-2)^2+3$$

$$\int \sqrt{(x-2)^2+3}\;dx$$

$$u = x-2$$

$$\int \sqrt{u^2+3}\;du$$

$$u = \sqrt{3}\tan\theta$$

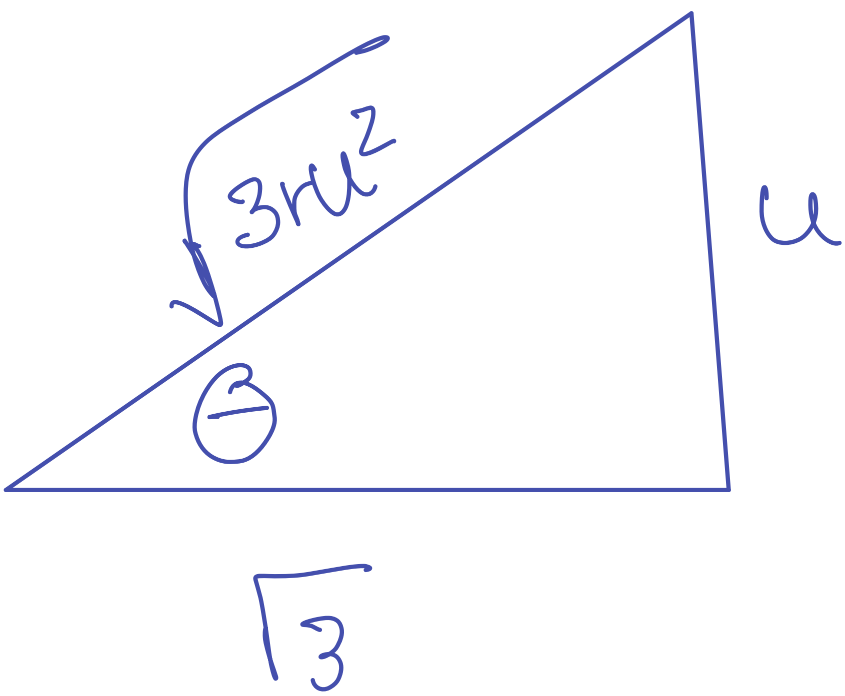
$$du = \sqrt{3}\sec^2\theta\;d\theta$$

$$\int \sqrt{3}\sec\theta \cdot \sqrt{3}\sec^2\theta\;d\theta$$

$$= 3\int \sec^3\theta \, d\theta$$

$$= \frac{3}{2}\sec\theta\tan\theta + \frac{3}{2}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$u = \sqrt{3}\tan\theta \qquad \tan\theta = u/\sqrt{3}$$




$$\sec\theta = \frac{\sqrt{3+u^2}}{\sqrt{3}}$$

$$= \frac{3}{2}\cdot\frac{\sqrt{3+u^2}}{\sqrt{3}}\cdot\frac{u}{\sqrt{3}} + \frac{3}{2}\ln\left|\frac{\sqrt{3+u^2}}{\sqrt{3}} + \frac{u}{\sqrt{3}}\right| + C$$

$$= \frac{3}{2}\,\frac{\sqrt{3+(x-2)^2}}{\sqrt{3}}\cdot\frac{(x-2)}{\sqrt{3}} + \frac{3}{2}\ln\left|\sqrt{\frac{3+(x-2)^2}{\sqrt{3}}} + \frac{x-2}{\sqrt{3}}\right|$$

$$+ C.$$