Trig integrals II

Ex: $\int \sin^2(x)\cos^2(x)\,dx = \int \left(\frac{1}{2}\sin(2x)\right)^2 dx$

$= \frac{1}{4}\int \sin^2(2x)\,dx = \frac{1}{4}\int \frac{1-\cos(4x)}{2}\,dx$

$= \frac{x}{8} - \frac{x}{32}\sin(4x) + C$

$\int \sin^m(x)\cos^n(x)\,dx$

- m or n odd: leave one factor of trig function, convert rest to other type.
- both even: repeatedly power reduce and use double angle as needed.

Hard!

Ex: $\int \cos^5 x \sin^3 x \, dx$

$$= \int \cos^5(x)\left(1-\cos^2(x)\right)\sin(x)\, dx$$

$$\int u^5(u^2-1)\, du$$

$$= \frac{1}{8}\cos^8(x) - \frac{1}{6}\cos^6(x) + C.$$

Next goal: $\int \tan^m x \sec^n x \, dx$

Ex: $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx$

$$= \tan x - x + C$$

Ex: $\int \tan^3 x \, dx$

$\int \tan x \, (\sec^2 x - 1) \, dx$

$= \frac{1}{2}\tan^2(x) - \ln|\sec(x)| + C$

Ex: $\int \tan^4 x \, dx$

$\int \tan^2(x)(\sec^2 x - 1) \, dx$

Ex: $\int \sec^3 x \, dx$

$$\int \sec^2 x \sec x \, dx$$

$u = \sec x \qquad du = \sec x \tan x$

$dv = \sec^2 x \qquad v = \tan x$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \underbrace{\tan^2 x}_{\sec^2 x - 1} \sec x \, dx$$

$$= \sec x \tan x + \int \sec x \, dx - \int \sec^3 x \, dx$$

$$2\int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x|$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$

$\int \tan^m x \sec^n x \, dx$:

- m odd, $n \geq 1$:

  factor out $\sec x \tan x$, convert tan to sec and u-sub

- n even:

  rip out $\sec^2 x$, convert secants to tan, u-sub

- m even, n odd:

  convert tan to sec. Hard!

Ex:

$$\int \tan^3 x \sec^5 x \, dx$$

$$= \int \tan x \sec x \tan^2 x \sec^4 x \, dx$$

$$\int (u^2 - 1) u^4 \, du$$

$$= \frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$$

Ex:

$$\int \tan^3 x \sec^4 x \, dx$$

$$\int \tan^3 x (\tan^2 x + 1) \sec^2 x \, dx$$

$$\int u^3(u^2+1)\,du$$

$$= \frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C.$$