Finishing up integration by parts:

Ex:

$$\int e^{\sqrt{x}}\,dx \qquad t = \sqrt{x}$$

$$dt = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2t}\,dx$$

$$dx = 2t\,dt$$

$$\int e^{\sqrt{x}}\,dx = \int 2te^t\,dt$$

$$u = 2t \qquad du = 2$$

$$dv = e^t \qquad v = e^t$$

$$\int 2te^t\,dt = 2te^t - \int 2e^t\,dt$$

$$= 2te^t - 2e^t + C$$

$$= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C.$$

Ex: $\int \frac{xe^x}{(x+1)^2}\,dx$

$u = xe^x \qquad du = (x+1)e^x$

$dv = \frac{1}{(x+1)^2} \qquad v = -\frac{1}{x+1}$

$$\int \frac{xe^x}{(x+1)^2}\,dx = -\frac{x}{x+1}e^x + \int e^x$$

$$= -\frac{x}{x+1}e^x + e^x + C$$

this one is rather hard!

Trigonometric Integrals

- Generally need knowledge of trig identities to simplify integrals w/ trig functions

Review of Trig identities:

- $\sin^2 x + \cos^2 x = 1$ — Pythagorean Identities
- $\tan^2 x + 1 = \sec^2 x$

- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$ — Double Angle
  $= 2\cos^2(x) - 1$
  $= 1 - 2\sin^2(x)$

- $\sin^2(x) = \frac{1 - \cos(2x)}{2}$ — Power Reduction
- $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

Note: there are many other trig identities like product to sum, sum to product, tangent double angle, etc. We won't bother with these, but they are other useful tools!

Common trig integrals:

- $\int \tan(x)\,dx = -\ln|\cos(x)| + C$

  (u-sub)

- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$

  (trick: mult. by $\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$)

- $\int \cot(x)\,dx = \ln|\sin(x)| + C$

- $\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$

Ex: $\int \sin^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx$

$= \frac{x}{2} - \frac{\sin(2x)}{4} + C$

Ex: $\int \sin^3(x)\,dx$

$= \int \sin(x)\cdot(1-\cos^2(x))\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$\int u^2 - 1\,du = \frac{1}{3}u^3 - u + C$

$= \frac{1}{3}\sin^3(x) - \sin(x) + C.$

Ex: $\int \sin^4(x)\,dx$

$$\int (\sin^2(x))^2\,dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

$$= \frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\,dx$$

power reduce again

$$= \frac{1}{4}\int 1 - 2\cos(2x) + \left(\frac{1+\cos(4x)}{2}\right) dx$$

$$= \frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$$