Integration by parts

Product rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$.

Integrate: $f(x)g(x) = \int f'(x)g(x) + f(x)g'(x)$

$u = f(x) \qquad du = f'(x)\,dx$

$v = g(x) \qquad dv = g'(x)\,dx$

$$uv = \int v\,du + u\,dv$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Integration by parts is a general technique for integrating products of functions.

Strategy:

- u should be easy to differentiate
- dv should be easy to integrate.

Ex: $\int x e^x \, dx$

$u = x \qquad du = 1$

$dv = e^x \qquad v = e^x$

$$\int x e^x \, dx = x e^x - \int e^x \, dx$$

$$= x e^x - e^x + C$$

$$= e^x (x - 1) + C.$$

Ex: $\int x^2 \sin(x) \, dx$

$u = x^2 \qquad du = 2x$

$dv = \sin x \qquad v = -\cos x$

$$\int x^2 \sin x \, dx = -x^2\cos(x) + \underline{\int 2x\cos x} \, dx$$

For $\int 2x\cos(x)\,dx$, $\quad u = 2x \quad dv = \cos(x)$

$du = 2 \quad v = \sin(x)$

$$\int 2x\cos(x)\,dx = 2x\sin(x) - \int 2\sin(x)\,dx$$

$$= \underline{2x\sin(x) + 2\cos(x) + C}$$

$$\int x^2\sin(x)\,dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C.$$

<u>Ex:</u> $\int \ln(x)\,dx$

$u = \ln(x)$ $\quad$ $du = 1/x$

$dv = 1$ $\quad$ $v = x$

$$\int \ln(x)\,dx = x\ln(x) - \int 1\,dx$$

$$= x\ln(x) - x + C$$

Ex: $\int e^x \sin(x)\,dx$

$u = e^x$ $\quad$ $dv = \sin(x)$

$du = e^x$ $\quad$ $v = -\cos(x)$

$$\int e^x \sin(x)\,dx = -e^x\cos(x) + \int e^x\cos(x)\,dx$$

$$\int e^x \cos(x)\,dx$$

$u = e^x \qquad du = e^x$

$dv = \cos(x) \qquad v = \sin(x)$

$$\int e^x\cos(x)\,dx = e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$\int e^x\sin(x)\,dx = -e^x\cos(x) + e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$2\int e^x\sin(x)\,dx = -e^x\cos(x) + e^x\sin(x)$$

$$\int e^x\sin(x)\,dx = \frac{1}{2}\left(-e^x\cos(x) + e^x\sin(x)\right) + C$$

Ex: $\int \sin(\ln(x))\,dx$

$t = \ln(x)$

$dt = 1/x\,dx \qquad dx = x\,dt$

$= e^t\,dt$

$$\int \sin(\ln(x))\,dx = \int e^t \sin(t)\,dt$$

use prev. problem