Recall: $\log_b(x) = \ln(x)/\ln(b)$.

So $\boxed{\dfrac{d}{dx}\log_b(x) = \dfrac{1}{\ln(b)\,x}}$.

Using inv. der formula, can

show $\boxed{\dfrac{d}{dx} b^x = b^x \ln(b)}$, so

$$\boxed{\int b^x \, dx = \frac{1}{\ln(b)} b^x + C.}$$

Ex.: $\dfrac{d}{dx} \pi^{5x-2} = 5\ln(\pi)\pi^{5x-2}$

$$\int x\, 3^{x^2} dx = \frac{1}{2}\int 3^u \, du = \frac{1}{2\ln(3)} 3^u + C = \frac{1}{2\ln(3)} 3^{x^2} + C$$

$u = x^2$ $\quad dx = \frac{1}{2x} du$

$du = 2x\, dx$

Logarithmic differentiation:

logarithms: products/quotients $\rightsquigarrow$ sums/differences

exponentiation $\rightsquigarrow$ multiplication idea: take log before diff. to make easier computation!

Ex: $f(x) = \sqrt{\dfrac{x(x+1)}{x^2+1}}$

$$\ln(f) = \frac{1}{2}\left[\ln(x) + \ln(x+1) - \ln(x^2+1)\right]$$

$$\frac{f'}{f} = \frac{1}{2}\left[\frac{1}{x} + \frac{1}{x+1} - \frac{2x}{x^2+1}\right]$$

Chain rule!

$$f' = \frac{1}{2} f \left[\frac{1}{x} + \frac{1}{x+1} - \frac{2x}{x^2+1}\right]$$

$$= \frac{1}{2}\sqrt{\frac{x(x+1)}{x^2+1}}\left[\frac{1}{x} + \frac{1}{x+1} - \frac{2x}{x^2+1}\right].$$

Much easier than older

methods!

Sometimes required to do this:

<u>Ex:</u> $f(x) = x^x$

this is <u>not</u> a comp. of functions

can't use chain rule.

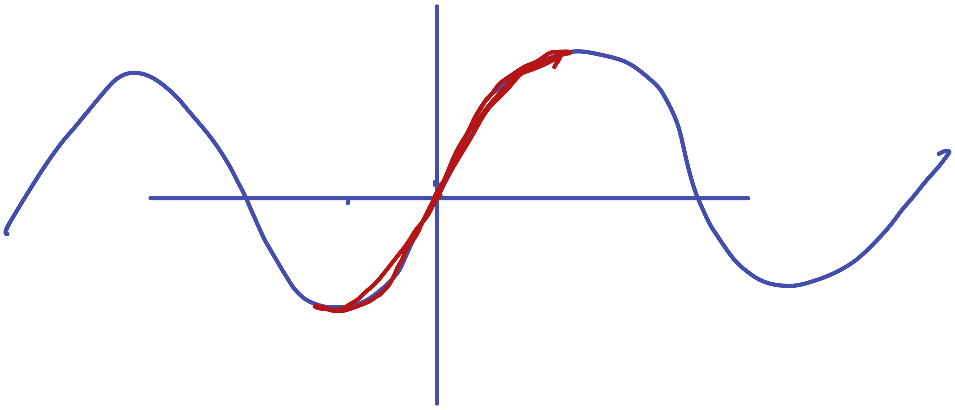
$\ln(f) = x\ln(x)$

$\frac{f'}{f} = \ln(x) + 1$

$f' = f(\ln(x)+1)$

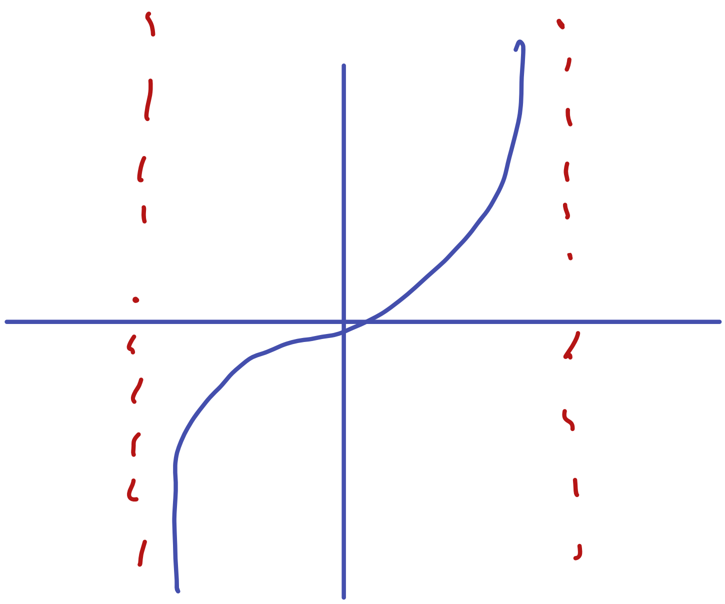
$= x^x(\ln(x)+1).$

Inverse Trig functions:



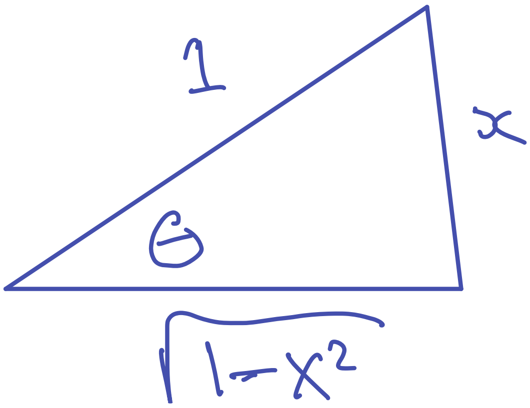
$\sin^{-1}(x)$: dom. $[-1, 1]$

range: $[-\pi/2, \pi/2]$



$\tan^{-1}(x)$: dom. $\mathbb{R}$

range: $(-\pi/2, \pi/2)$



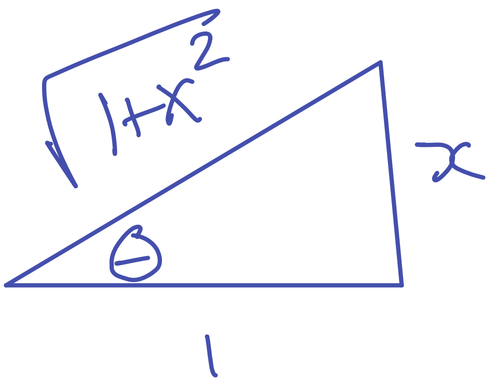
$\sin(\theta) = x$

$\theta = \sin^{-1}(x)$

$$\sin(\sin^{-1}(x)) = x$$

$$\underbrace{\cos(\sin^{-1}(x))}_{\sqrt{1-x^2}} \cdot (\sin^{-1}(x))' = 1$$

$$\boxed{\begin{aligned} \frac{d}{dx}\sin^{-1}(x) &= 1/\sqrt{1-x^2} \\ \int \frac{1}{\sqrt{1-x^2}}\,dx &= \sin^{-1}(x) + C \end{aligned}}$$



$\theta = \tan^{-1}(x)$

$$\tan(\tan^{-1}(x)) = x$$

$$\underbrace{\sec^2(\tan^{-1}(x))}_{1+x^2}\,(\tan^{-1}(x))' = 1$$

$$\boxed{\begin{aligned} &\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \\ &\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C \end{aligned}}$$

less important:

$$\boxed{\begin{aligned} &\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \\ &\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C \end{aligned}}$$

derivatives of inverse "co"-functions are just negatives of these.

Ex: $\int \frac{1}{\sqrt{4-9x^2}}\,dx = \int \frac{1}{2\sqrt{1-\frac{9}{4}x^2}}\,dx$

$u = \frac{3}{2}x$

$du = \frac{3}{2}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{3}\sin^{-1}(u) + C$

$= \frac{1}{3}\sin^{-1}\left(\frac{3}{2}x\right) + C$