Math 31B
Homework 7
Due Thursday, March 16, 2023

Instructions: Show all work that was done to arrive at your answer. Answers with no work will not receive credit. Use full sentences when necessary.

Do the following problems from each section of the textbook:

- 11.6: 12,19,22,26,28,38,42,46,55

- 11.8: 7,12,14,16,20

Do the following additional problem:

1. The *Fibonacci sequence* F_n is defined by $F_0 = F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. The first few terms of the Fibonacci sequence are $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$. Define

$$F(x) = \sum_{n=0}^{\infty} F_n x^n = 1 + x + 2x^2 + 3x^3 + 5x^4 + \ldots$$

Since $F(x)$ is a power series, it has some interval of convergence I. Your goal will be to find a function whose Maclaurin series is $F(x)$ inside I.

   (a) Set $S = 1 + x + 2x^2 + 3x^3 + 5x^4 + \ldots$. Write down xS and $x^2 S$. What is the coefficient of a general term x^n in S? In xS? In $x^2 S$?

   (b) Show that $S - xS - x^2 S = 1$, and therefore deduce that $F(x) = \frac{1}{1-x-x^2}$ for all $x \in I$. (*Hint: what is the coefficient of a general term x^n of the left hand side?*)

   (c) (bonus) Use partial fractions to find the Maclaurin series of $\frac{1}{1-x-x^2}$ in a different way. Use this to help you find an explicit formula for F_n.