

Math 31B
Homework 6
Due Thursday, March 2, 2023

Instructions: Show all work that was done to arrive at your answer. Answers with no work will not receive credit. Use full sentences when necessary.

Do the following problems from each section of the textbook:

- 11.3: 12,70,77
- 11.4: 3,24,26,27
- 11.5: 8,10,18,28,40,48,51

Do the following additional problems:

1. Recall the gamma function, $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$. On homework 4, you showed using integration by parts that $\Gamma(x)$ converges for $x > 0$. On this homework, you'll give a better proof that $\Gamma(x)$ converges for $x > 0$.
 - (a) Show that for any $x > 0$, the integral test applies to $\int_0^\infty e^{-t} t^{x-1} dt$, so that $\int_0^\infty e^{-t} t^{x-1} dt$ and $\sum_{n=0}^\infty e^{-n} n^{x-1}$ have the same behavior.
 - (b) Use the ratio test to deduce that for any $x > 0$, $\sum_{n=0}^\infty e^{-n} n^{x-1}$ converges.
2. For each of the following series, state if they converge (absolutely or conditionally, if applicable) or diverge, and which test you would use to determine this. If you use a comparison test, state which function you would compare with. If you know how to compute the value of the sum, you should write what it is. You do **not** need to write the proofs of converge/divergence, but I encourage you to do so if you have time!
 - (a) $\sum_{n=0}^\infty \frac{\pi^{2n}}{e^{3n}}$
 - (b) $\sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^2}$
 - (c) $\sum_{n=0}^\infty \frac{\cos(n)}{n^{3/2}}$
 - (d) $\sum_{n=0}^\infty \frac{n^4}{n!}$
 - (e) $\sum_{n=1}^\infty \frac{1}{n\sqrt{n+\ln(n)}}$
 - (f) $\sum_{n=1}^\infty \ln\left(\frac{n}{n+1}\right)$