

Math 31B  
Homework 1  
Due Thursday, January 19, 2023

**Instructions:** Show all work that was done to arrive at your answer. Answers with no work will not receive credit. Use full sentences when necessary.

Do the following problems from each section of the textbook:

- 7.1: 26,29,30,73,76
- 7.2: 18,22,34
- 7.3: 32,66,72,89,98,103

Do the following additional problems:

1. Which is larger,  $\pi^e$  or  $e^\pi$ ? In this problem, you'll use calculus to figure this out *without* a calculator.
  - (a) Let  $f(x) = x^{1/x}$ . What is the largest domain on which  $f(x)$  can be defined? Show using calculus that  $f(x)$  has a unique maximum value on this domain, and find the  $x$ -value where it happens. What is the range of  $f(x)$ ?
  - (b) Which is larger:  $f(\pi)$  or  $f(e)$ ? Use this to determine which of  $\pi^e$  and  $e^\pi$  is larger. (*Hint: find a common exponent to raise both  $f(\pi)$  and  $f(e)$  to.*)
2. (Derivative patterns)
  - (a) Let  $f(x) = xe^x$ . Write down  $f'(x), f''(x), f'''(x), f^{(4)}(x)$ . Do you see a pattern? What is  $f^{(n)}(x)$  for an arbitrary integer  $n \geq 0$ ? (*Note: by convention,  $f^{(0)}(x) = f(x)$ .*)

For an integer  $n$ , the *factorial* of  $n$ , denoted  $n!$ , is defined by the recursive product  $n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1$  for  $n \geq 1$  and  $0! = 1$ . For example,  $1! = 1$ ,  $2! = 2 \cdot 1 = 2$ , and  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ .

- (b) Let  $f(x) = \ln(1+x)$ . Compute  $f'(x), f''(x), f'''(x), f^{(4)}(x)$ . Do you see the pattern? What is  $f^{(n)}(x)$  for arbitrary integer  $n \geq 1$ ?