Math 31B
Integration and Infinite Series

Practice Midterm II

Directions: Do the problems below. You have 50 minutes to complete this exam. You may use a basic calculator without graphing or symbolic calculus capabilities. Show all your work. Write full sentences when necessary.

Name: _____

UID: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

Formula Sheet

Derivatives

- $\frac{d}{dx} b^x = b^x \ln(b)$
- $\frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)}$
- $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$
- $\frac{d}{dx} \sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integrals

- $\int u\,dv = uv - \int v\,du$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int \tan(x)\,dx = \ln|\sec(x)| + C$
- $\int \cot(x)\,dx = \ln|\sin(x)| + C$
- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$

Numerical Integration

- $M_N = \Delta x(f(c_1) + f(c_2) + \ldots + f(c_N))$, c_i midpoint of $[x_{i-1}, x_i]$.
- $T_N = \frac{1}{2}\Delta x(y_0 + 2y_1 + y_2 + \ldots + 2y_{N-1} + y_N)$, $y_i = f(x_i)$.
- $S_N = \frac{1}{3}\Delta x(y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-2} + 4y_{N-1} + y_N)$, $y_i = f(x_i)$.
- $\text{Error}(M_N) \leq \frac{K_2(b-a)^3}{24N^2}$
- $\text{Error}(T_N) \leq \frac{K_2(b-a)^3}{12N^2}$
- $\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$

K_2 and K_4 are upper bounds of $|f''(x)|$ and $|f^{(4)}(x)|$ on the interval $[a, b]$ respectively.

1. (10 pts.) For each of the following series, state if they converge (conditionally or absolutely, if applicable) or diverge. Justify your answers carefully.

   (a) (5 pts.) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{3^n \cdot n^3}{\sqrt[3]{n^n}}$

   (b) (5 pts.) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \cos\left(\frac{1}{n^3}\right)$

2. (10 pts.) For each of the following series, state if they converge (conditionally or absolutely, if applicable) or diverge. Justify your answers carefully.

   (a) (5 pts.) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)\ln(\ln(n))}$

   (b) (5 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{1+2^n}$

3. (10 pts.) For each of the following series, state if they converge (conditionally or absolutely, if applicable) or diverge. Justify your answers carefully.

   (a) (5 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{2n}}{3^n + 3^{3n}}$

   (b) (5 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+3}$

4. (10 pts.) Suppose $f(x)$ is a function such that $0 \leq f''(x) \leq 2$ for all $x \in [0,1]$. Suppose you wanted to estimate $\int_0^1 f(x)\,dx$, and you have determined $f(x)$ at the following points:

x	$f(x)$
0	0
1/8	1
1/4	1/2
3/8	2
1/2	3/4
5/8	3
3/4	4
7/8	2/3
1	1

   (a) (2 pts.) Compute M_4.

   (b) (3 pts.) Find upper and lower bounds of $\int_0^1 f(x)\,dx$.

   (c) (5 pts.) Find a value of N such that M_N approximates $\int_0^1 f(x)\,dx$ to within 5 decimal places.