Math 31B
Integration and Infinite Series

Practice Midterm

Directions: Do the problems below. You have 50 minutes to complete this exam. You may use a basic calculator without graphing or symbolic calculus capabilities. Show all your work. Write full sentences when necessary.

Name: _____

UID: _____

Question	Points	Score
1	10	
2	12	
3	12	
4	8	
Total:	42	

Formula Sheet

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$
- $\sin^2(x) = \frac{1-\cos(2x)}{2}$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}$

Derivatives

- $\frac{d}{dx}(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
- $\frac{d}{dx}b^x = b^x \ln(b)$
- $\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$
- $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integrals

- $\int u\,dv = uv - \int v\,du$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int \tan(x)\,dx = \ln|\sec(x)| + C$
- $\int \cot(x)\,dx = \ln|\sin(x)| + C$
- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$

1. (10 pts.) Compute the following:

   (a) (5 pts.) $\dfrac{d}{dx}(2^x+1)^{\ln(2^x+1)}$

   (b) (5 pts.) $\lim\limits_{x\to 0^+} xe^{1/x^2}$

2. (12 pts.) Compute the following:

   (a) (4 pts.) $g'(\pi/4)$, where $g(x) = f^{-1}(x)$ and $f(x) = \ln(x) + \tan^{-1}(x)$.

   (b) (8 pts.) $\displaystyle\int \frac{e^x}{(1+e^{2x})^{3/2}}\,dx$

3. (12 pts.) Compute the following integrals:

   (a) (6 pts.) $\displaystyle\int \frac{1}{2}\tan(\sqrt{x})\sec(\sqrt{x})\,dx$

   (b) (6 pts.) $\displaystyle\int_0^{\pi/3} \tan^5(x)\sec^5(x)\,dx$

4. (8 pts.) Compute the following integral:

$$\int \frac{2x^3 + 2x^2 - 2x + 1}{x^2(x-1)^2}\, dx$$

(Challenge) Compute the following integral:

$$\int \sin^{-1}(\sqrt{x})\,dx$$

Note: this problem is too hard for a midterm, but it's great practice!