Math 31B
Integration and Infinite Series

Midterm II

Directions: Do the problems below. You have 50 minutes to complete this exam. You may use a basic calculator without graphing or symbolic calculus capabilities. Show all your work. Write full sentences when necessary. If you need more space for scratch work, use the extra pages provided. **DO NOT WRITE ON THE BACK OF THE PAGE**.

Name: _____

UID: _____

Question	Points	Score
1	12	
2	12	
3	8	
4	8	
Total:	40	

Formula Sheet

Derivatives

- $\frac{d}{dx} b^x = b^x \ln(b)$
- $\frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)}$
- $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$
- $\frac{d}{dx} \sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integrals

- $\int u\,dv = uv - \int v\,du$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int \tan(x)\,dx = \ln|\sec(x)| + C$
- $\int \cot(x)\,dx = \ln|\sin(x)| + C$
- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$

Numerical Integration

- $M_N = \Delta x(f(c_1) + f(c_2) + \ldots + f(c_N))$, c_i midpoint of $[x_{i-1}, x_i]$.
- $T_N = \frac{1}{2}\Delta x(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{N-1} + y_N)$, $y_i = f(x_i)$.
- $S_N = \frac{1}{3}\Delta x(y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-2} + 4y_{N-1} + y_N)$, $y_i = f(x_i)$.
- $\text{Error}(M_N) \leq \frac{K_2(b-a)^3}{24N^2}$
- $\text{Error}(T_N) \leq \frac{K_2(b-a)^3}{12N^2}$
- $\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$

K_2 and K_4 are upper bounds of $|f''(x)|$ and $|f^{(4)}(x)|$ on the interval $[a, b]$ respectively.

1. (12 pts.) For each of the following series, state if they converge (conditionally or absolutely, if applicable) or diverge. Justify your answers carefully.

   (a) (6 pts.) $\displaystyle\sum_{n=3}^{\infty}(-1)^{n+1}\frac{n^{2023}\cdot 2023^n}{(2n+1)!}$

   (b) (6 pts.) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n}-1+2\ln(n)}{3n(\sqrt{n}+4)}$

2. (12 pts.) For each of the following series, state if they converge (conditionally or absolutely, if applicable) or diverge. Justify your answers carefully.

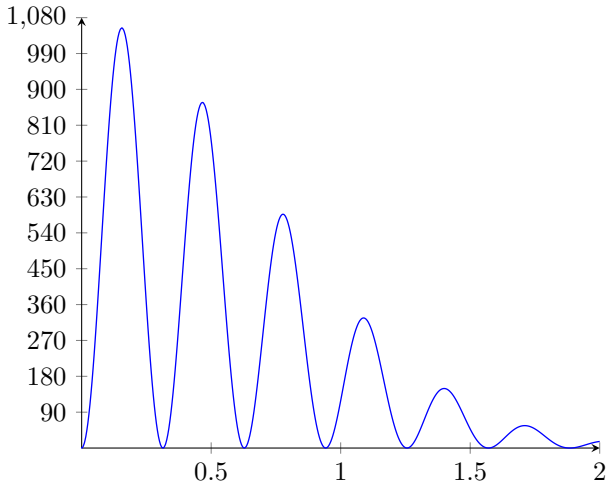
   (a) (6 pts.) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^{n-1}\cos(n)}{2^{3n}+(3n)^2}$

   (b) (6 pts.) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^n}}{n^{\sqrt{n}}}$

3. (8 pts.) Determine if the following infinite series converges (conditionally or absolutely, if applicable) or diverges. Justify your answer carefully.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)}}$$

4. (8 pts.) Below is a graph of $f^{(4)}(x)$ for some function $f(x)$ on the interval $[0, 2]$.



(a) (5 pts.) Find, with justification, a value of N such that the N-th Simpson's rule estimate S_N approximates $\int_1^{1.5} f(x)\,dx$ to within 4 decimal places. Give an upper bound for $\int_1^{1.5} f(x)\,dx$ in terms of S_N.

(b) (3 pts.) Suppose you also know that $0 \leq f(x) \leq e^{-x}$ for $x \geq 1.5$. Explain why $\int_1^{\infty} f(x)\,dx$ converges and give an upper bound for $\int_1^{\infty} f(x)\,dx$.

Scratch Paper

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