

Math 31B
Integration and Infinite Series
Midterm 1

Directions: Do the problems below. You have 50 minutes to complete this exam. You may use a basic calculator without graphing or symbolic calculus capabilities. Show all your work. Write full sentences when necessary. Simplify all terms involving expressions like $\cos(\sin^{-1}(x))$, etc. If you need more space for scratch work, use the extra pages provided. **DO NOT WRITE ON THE BACK OF THE PAGE.**

Name: _____

UID: _____

Question	Points	Score
1	12	
2	12	
3	8	
4	8	
Total:	40	

Formula Sheet

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $\cot^2(x) + 1 = \csc^2(x)$
- $\sin(2x) = 2 \sin(x) \cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$
- $\sin^2(x) = \frac{1 - \cos(2x)}{2}$
- $\cos^2(x) = \frac{1 + \cos(2x)}{2}$

Derivatives

- $\frac{d}{dx}(f^{-1})(x) = \frac{1}{f'(f^{-1}(x))}$
- $\frac{d}{dx} b^x = b^x \ln(b)$
- $\frac{d}{dx} \log_b(x) = \frac{1}{x \ln(b)}$
- $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$
- $\frac{d}{dx} \sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integrals

- $\int u \, dv = uv - \int v \, du$
- $\int \frac{1}{x} \, dx = \ln |x| + C$
- $\int \tan(x) \, dx = \ln |\sec(x)| + C$
- $\int \cot(x) \, dx = \ln |\sin(x)| + C$
- $\int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + C$
- $\int \csc(x) \, dx = \ln |\csc(x) - \cot(x)| + C$

1. (12 pts.) Compute the following:

(a) (6 pts.) $\frac{d}{dx} \left(\frac{x \cdot (\sqrt[4]{x+1})^3}{e^{\sin^{-1}(x)} \cdot (x^3 + 3)} \right)$

(b) (6 pts.) $\lim_{x \rightarrow 0} \cos(x)^{\csc(x)}$

2. (12 pts.) Compute the following integrals:

(a) (6 pts.) $\int \sin^{2023}(x) \cos^5(x) dx$

(b) (6 pts.) $\int \frac{x^2}{(4+x^2)^2} dx$

3. (8 pts.) Compute the following integral:

$$\int \frac{3x^3 + 2x^2 + 3}{x^2(x^2 + 1)} dx$$

4. (8 pts.) Compute the following integral:

$$\int \frac{\ln(x^4 + 1)}{x^3} dx$$

Scratch Paper

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