Quiz 3 Solutions

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1. Prove that there is no integer $x \in \mathbb{Z}$ for which $10x \equiv 7 \bmod 35$.

Solution: Suppose otherwise, that there is such an x. Then $10x \equiv 7 \bmod 35$ says $10x+35y = 7$ for some integer y. However, 5 divides the left hand side, while $5 \nmid 7$, a contradiction. Therefore, no such x exists.

2. Compute $\gcd(253, 649)$, and find a solution (x, y) to $253x + 649y = \gcd(253, 649)$.

Solution: Run the Euclidean algorithm:

$$649 = 2 \cdot 253 + 143$$

$$253 = 1 \cdot 143 + 110$$

$$143 = 1 \cdot 110 + 33$$

$$110 = 3 \cdot 33 + 11$$

$$33 = 3 \cdot 11$$

so that $\gcd(253, 649) = 11$. Running the back-substitution, $11 = 110 - 3 \cdot 33 = 4 \cdot 110 - 3 \cdot 143 = 4 \cdot 253 - 7 \cdot 143 = 18 \cdot 253 - 7 \cdot 649$, so that $(18, -7)$ is a solution.

3. For all $n \in \mathbb{N}$, let $S(n)$ denote the sum of the digits of n. Prove for all $n \in \mathbb{N}$ that $S(n) \equiv n \bmod 9$.

Solution: Write $n = a_d a_{d-1} \dots a_0$, where a_i are the digits of n. By definition, we see that $n = a_d 10^d + a_{d-1} 10^{d-1} + \dots + a_0$. Since $10 \equiv 1 \bmod 9$, this says $n \equiv a_d + a_{d-1} + \dots + a_0 \equiv S(n) \bmod 9$ as desired.