Modulus Arithmetri

$$a \equiv b \mod a \Leftrightarrow n \mid a - b$$
.
Drive algorithm Socys for any $a \in \mathbb{Z}$,
 $\exists q, r \mid a \equiv nq + r, \quad 0 \leq r \leq n - 1$.
So $a \equiv 0, 1, -, n - 1 \mod n$ are the
only possibilities
These correspond to sets
 $\{-n, -n + 1, 1, n + 1, 2n - 1, - -1\} = [n - 1]$
 $\{-n, -R + 1, n - 1, 2n - 1, - -1\} = [n - 1]$

$$\mathbb{Z}[n\mathbb{Z} = \{ Io1, ..., [n-1] \}$$

azy mode
$$C \equiv d \mod n$$

at $k \equiv b \neq d \mod n$
ac $\equiv b d \mod n$

Equations like the above.

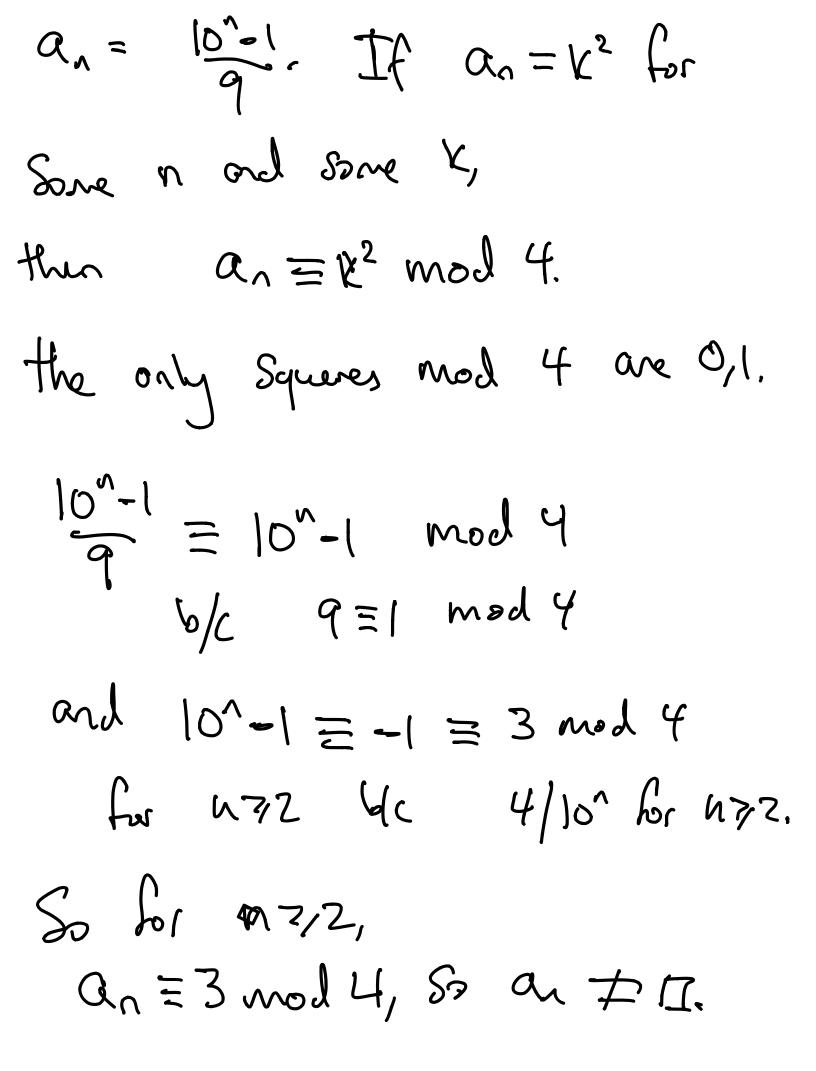
$$ax \equiv b \mod a$$
 $ax \equiv b + nk$
for some k
 $ax \equiv b \mod a$ for some k
 $ax + ny \equiv b$
Then: $ax \equiv b \mod a$ has a solution
iff $(a,n) \equiv d \mid b$.
If $X_0 \mod a$ is one solution,
then we have exactly $d = 1$ solutions
 $Mod n$, and they ar
 $X_0 \mod n$, $X_0 + \frac{1}{4} \mod n$, ..., $X_0 + (d+) \frac{1}{4} \mod n$

Cor: axel mod n has a sola

^_

10x = 34 mod 42 for some K. 10x = 34 + 42k5x = 17+21k for Some k. 5x + 21y = 17.21 - 5.4 = 15. (-68) + 21.17 = 17 $\Rightarrow 10.(-68) + 21 - 34 = 34$ ⇒ 10. (-68) = 34 mod 42 So X, = -68 = 16 mod 42 11 one Solution, and the Other Solution is

16+21 = 37 mod 42. X = 16, 37 mod 42. Sə Miscellaneous Problems Prove that more of 1,11,111,1111,.... are perfect Squares except the first term. Proof: the nth term in the Sequence is



Q1=1 is a Square FI

Prove that 15x²-7y²=9 has no intères Solutions.

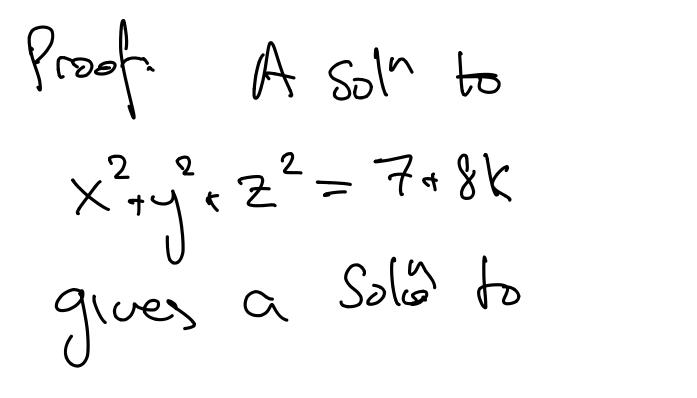
Proof. Sufficient to find on a sit. $15x^2 - 7y^2 = 9 \mod n$ has no Solution Note that 3/18x2, and 3/9 $\implies 3(7y^2, \implies 3)y.$ y = 3y

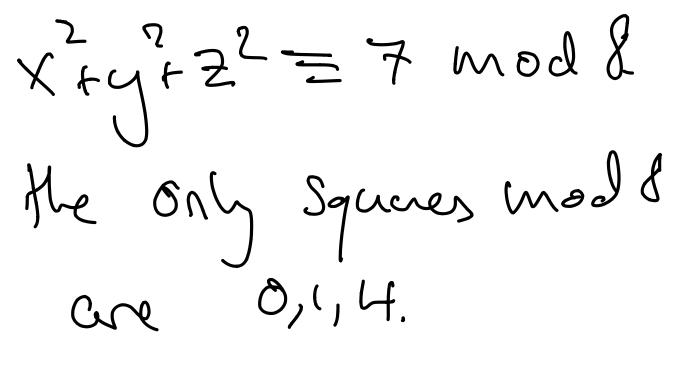
 $15x^{2} - 7.9y^{2} = 9$ 9 6342 9/9 $\Rightarrow 9|15x^2 \\ \Rightarrow 3|x,$ $X = 3x_1$ $15.9 \times ^{7}_{,} - 7.9 y_{,}^{2} = 9$ $15x_1^2 - 7y_1^2 = 1.$ So were shown a solution $15x^2 - 7y^2 = 9$ gives a solution to $|5x^2 - 7y^2 = 1$ Reducing mod 3, $-7y^2 \equiv 1 \mod 3$

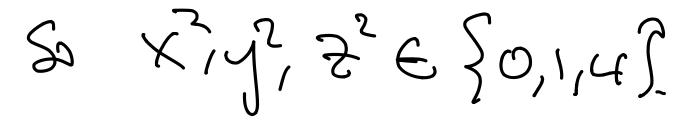
 $2y^2 \ge 1 \mod 3$ = y = 2 mod 3. this has no Sol, ble only Squeres mod 3 are 0,1. $\implies 15x^2 - 7y^2 = 1$ has no $Solu \implies (Sx^2 - 7y^2 = 9)$ here no Solu

Prove that no integer of the form 7+8% is a Sum of

3 Squres.







Brute force to check that none of the 27 possibilities work of