Problems

<u>Ex</u>: Prove that the product of K consecutive integers is divisible by K !.

Troop. Let the integers be n, n-1, n-2, ..., n-(x-1). This graduat is n(n-1)(n-2) - ... (n - (K-1))Note that this expression equele n! $\frac{n!}{n!} = \binom{n}{k}$ $(n-\kappa)$ $= K! \binom{n}{k}$, so we're done

Ex. M-n is drussible by 6 for all N, b/c view = (n-i)n(nri) is the product of 3 consecutie Integers. Ex: Prove that ((n+i)!+1, n!+1) = 1.

 $\frac{lroof}{let} d = gcd((nti)!, tl, n!, tl).$ Since dis a common durson, d (n+i)!+1 and d/n!+1. $S_0 d ((n_1)_{+1}) - (n_{+1})$

$$(n+i)! - n! = n \cdot n!$$
So $d | n \cdot n!, and $d | n! + i,$

$$\Rightarrow d | n \cdot n! + n and d | n \cdot n!$$
So $d | n. From d | n and$

$$d | n! + i, we see that$$

$$d | n \cdot (n-i)! = n! and d | n! + i$$

$$\Rightarrow d | 1 \Rightarrow d = 1$$$

Ex: Let a,b be integers with (a,b) = 1. Prove that $\left(a^2,b^2\right)=1.$ Kroof: By Bezont's lemmer, there are Integers X and y Such that

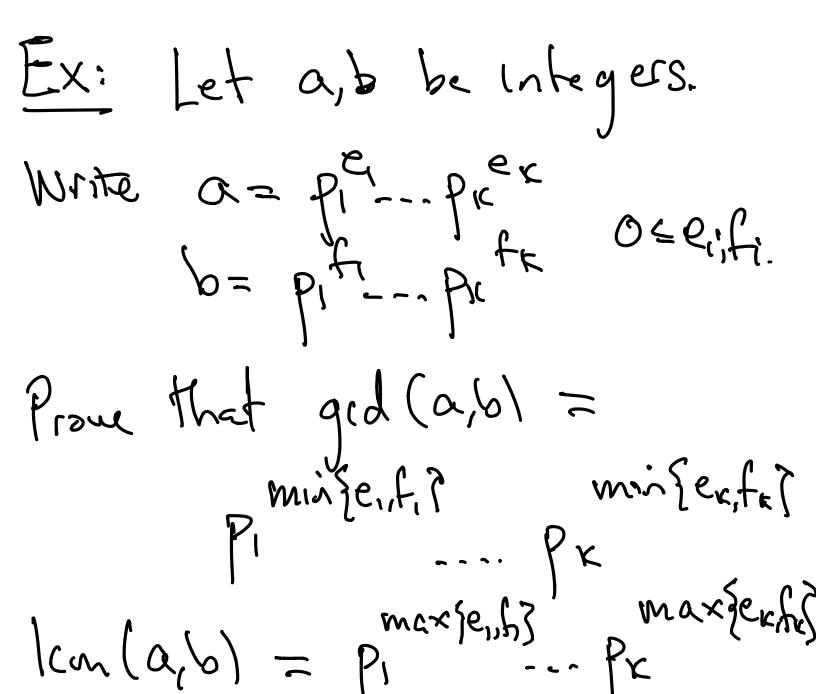
ax+by=1. We want to Show: $a^2u + b^2v = 1$ for Some integers u, v.

First try: Squere both Sides.

 $(\alpha \times t by)^2 = 1$ $a^2x^2 + 2abxy + b^2y^2 = 1$ this doesn't help ble of the middle term. Second by: Culoe both Sides. $(axtby)^3 = 1$ $a^{3}x^{3} + 3a^{2}x^{2}by + 3axby + by = 1.$ $a^{2}(ax^{3}+3x^{2}by)+b^{2}(3axy^{2}+by^{3})=1$ these are integers

So voe'ne found
$$u, v u u a^{2} u + b^{2} v = 1, =) (a^{2}, b^{2}) | 1$$

$$= (a^{2}, b^{2}) = 1$$



Proof: Let d'be a common divisor of a and b. By unique factorization, write d= pi --- pr for some Osti. Since da, we know that ti Lei for all i. Similarly, d/b, so ti 2 fi ho all i. $S_0 \quad t_i \leq \min\{e_i, f_i\}$

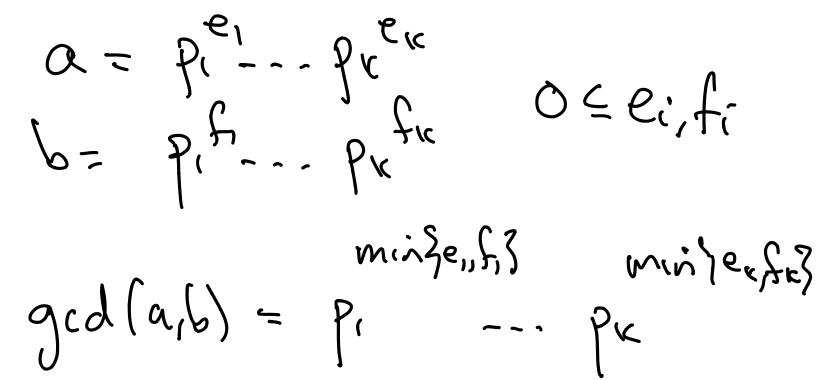
Note that for any integr Of the form pt --- pret w ostis minfeifij that this actually is a dronsor of a and b b/c can malhpy by missing proves to get either a or b as needed. 10 get largent Such divisor, maximize ti. So $gcd = p_1$ \dots p_k r

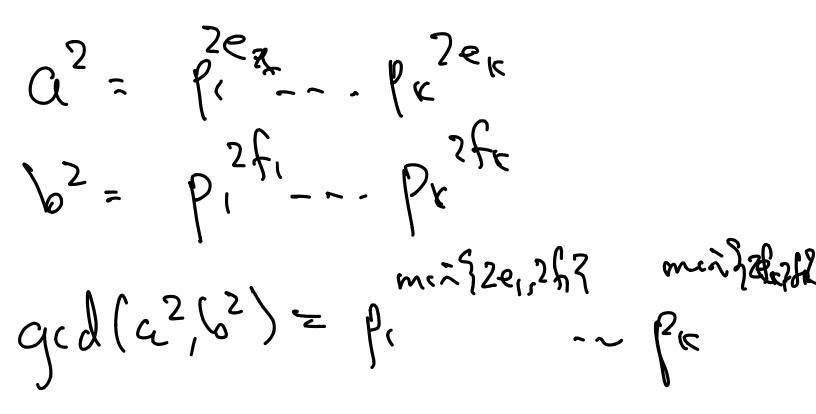
To get the Icm, let m be a multiple st a coul b. So alm and blm. $M = P_1 - P_1 k$ by daugue $0 \leq S_i$. Since a|m, $e_i \leq s_i$ for all blue $f_i \leq s_i$ i So Siz, max {ci,fi], Again, note that for

ang Si Zi maxgeifi? that PI-... PK is a (some multiple, So we get the least Common mult by minimizing each exponent. maxfei,hj mexjectif lcon(a,b) = pi -- pr F

We can use this result to give an alternate

proof of the previous problem:





$$\frac{S_{1}}{2} = \frac{n}{2} = \frac{n}{2} = \frac{n}{3} = \frac{n}{3} = \frac{1}{3} =$$

We know that I has to be divisible by 2,3,5. Let's n cusing only Construct Such an these proves. n = 23fgfor some e, hg 7, 1 e has be odd. Since N=2K² e has to be Since n= 3m², divisible by 3 e has to be Since N = JQJ, divisible by 5.

So e has to be an odd maltiple of 15, so can take e=15,

Doiny this for fand gi of has to be even of has to be of the form 38+1 is duriallate by 5 • f f' = 10

• g has to be even • g dwidble by 3 • g of the form 5km

g = G

We can talke $N = 2 \cdot 3 \cdot 5$