## Worksheet 4

Problems marked with a (\*) are "key results".

- 1. Carry out the Euclidean algorithm and run it backwards to find x, y such that ax + by = gcd(a, b) for the following pairs of integers (a, b):
  - (a) (909, 489)
  - (b) (1819, 3587)

Can you find other values of x, y that work?

- 2. Find an integer solution to 37x + 47y = 103. (*Hint: what is gcd*(37,47)? *How might this help*?)
- 3. Compute the greatest common divisor and least common multiple of several pairs of integers, like (12, 8), (15, 16), and (72, 111). How do the gcd and lcm appear to be related?
- 4. Prove the following statements:
  - (a) (\*) Let  $a, b \in \mathbb{Z}$  both non-zero. If  $d \mid a$  and  $d \mid b$ , for some integer d, then  $d \mid \gcd(a, b)$ .
  - (b) Let  $a, b \in \mathbb{Z}$  both non-zero. If  $a \mid c$  and  $b \mid c$  for some integer c, then  $gcd(a, b) \mid c$ .
- 5. (\*) Let  $a, b \in \mathbb{Z}$  both non-zero. Let a = a'd and b = b'd for some integers a', b' and  $d = \gcd(a, b)$ . Prove that  $\gcd(a', b') = 1$ .
- 6. (\*) Let  $a, b, c \in \mathbb{Z}$ , and suppose that (a, b) = 1. Prove that if  $a \mid bc$ , then  $a \mid c$ .