Worksheet 3

Problems marked with a ($*$) are "key results".

1. Introduce yourself to your group if you haven't done so yet :)

2. Come up with a definition for the "least common multiple" of two integers $a, b \in \mathbb{Z}$. We'll denote this by $\operatorname{lcm}(a, b)$, or $[a, b]$.

3. Carry out the Euclidean algorithm and verify the following:

   (a) $\gcd(519, 163) = 1$.

   (b) $\gcd(8602, 4278) = 46$.

   Can you find integers x, y such that $519x + 163y = 1$?

4. Which pairs of two digit numbers have the longest number of steps in the Euclidean algorithm? Three digit numbers? Do you have any conjectures?

5. The goal of this problem is to sketch a proof of the Euclidean algorithm.

   (a) ($*$) Show that there cannot be an infinite, strictly decreasing sequence of non-negative integers.

   (b) ($*$) Let $a, b \in \mathbb{Z}$ with $a, b \neq 0$. Using the extended division algorithm from homework 2, write

   $$a = bq + r, \quad 0 \leq r < |b|$$

   for some integers q, r. Prove that $\gcd(a, b) = \gcd(b, r)$.

   (c) Explain how you would combine these two results to give a complete proof of the Euclidean algorithm.