This is false!

$$V = F[X]$$

 $B_i = \frac{2}{2} [tX_i^i X tX_i^i \dots]$
Claimin it's not possible to form
a basis for V with a varter
from each B_i .

ai, bi.

Suppose $1 = \sum_{i=1}^{n} C_i (x^{a_i} + x^{b_i})$ i = 1for some C_i , all $c_i \neq 0$.

Note that on of the exponents
that appears in the snon must be
$$O$$
:
otherwise, plug in $K=0$ to RHS
to get $1=0$.

Also note if an exponent appears in one term, it must appear is another in order to concel out.

2n=1 ≤ n-1, So ≤ n-1 non-zero exponents that con

appear. Exact team in Sum comes from Bai or Bbi Pidgeon hole Suys Hure are two terms in the Sun coming from Some basis The

S. $T: V \rightarrow V$ $dim_{e} V = 6$ $W_{1}, W_{2} \subset V$

din _e w₁ = din _e w₂ = 4 ding WINWZ = 2. $M_{T_{1}} = (x-i)(x-2)(x-3)$ $M_{\overline{1}2} = \chi(\chi - 1)$

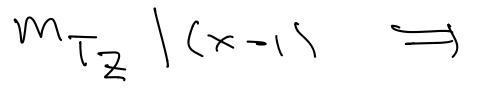
a.) m_{TZ}/M_T

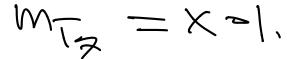
becanse

M72/M52

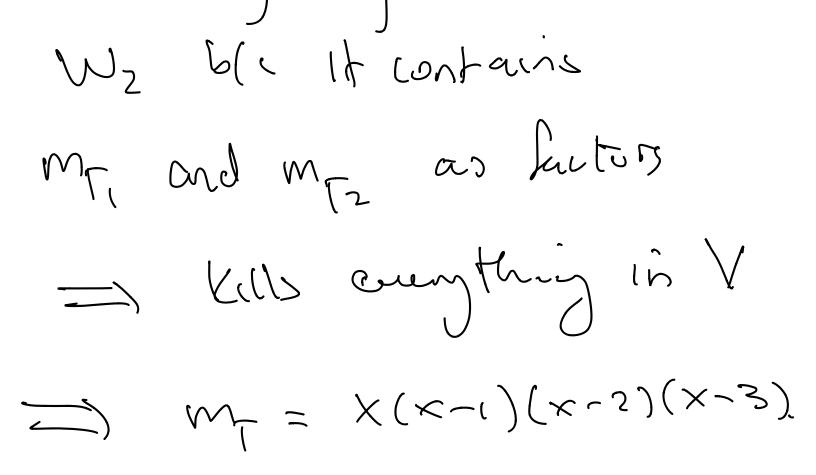
ZCW, ZCWZ

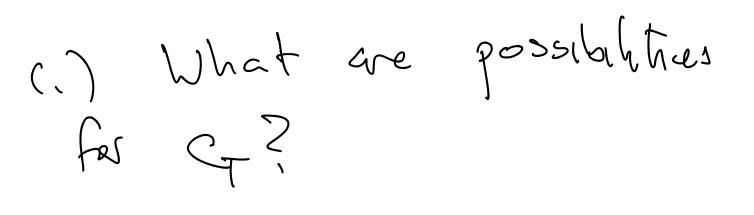


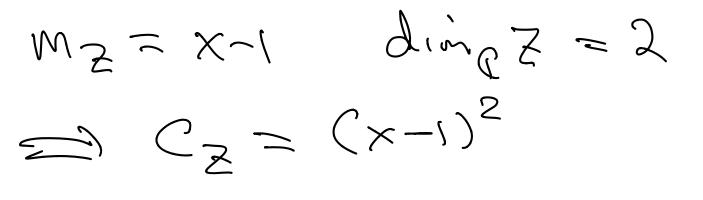


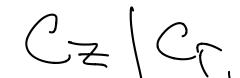


 $= \int_{Z} \int_{Z} = 0$ (z = 1 $w, c \vee$ $m_{\overline{1}}/m_{\overline{1}}$ 6.1 $w_2 \subset \vee$ miz Im X(x-1)(x-2)(x-3) [m_t \longrightarrow Note that $W, + W_2 = V$ for diminsion reasons. X(x-1)(x-2)(x-3) Kills everything in W, and







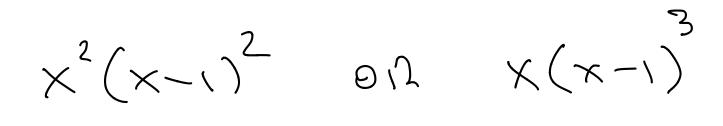


$C \leq (C^{2})$

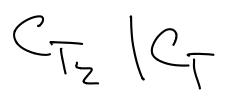






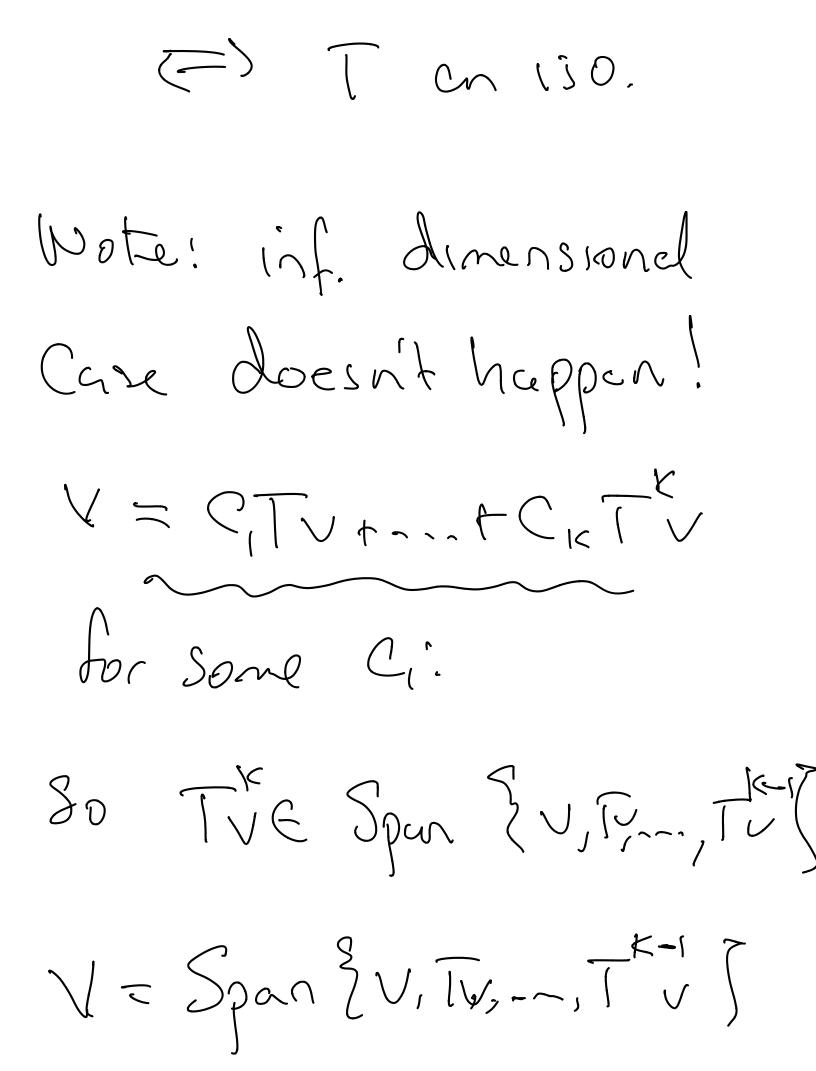






 $\sum_{i} C_{i} = \chi^{2}(\chi_{-1})^{2}(\chi_{-2})(\chi_{-3})$ $C_{T} = X(X-1)^{3}(X-2)(X-3)$ $d.) \quad m_{T} = \chi(\chi_{-1})(\chi_{-2})(\chi_{-3})$ distinct lineer factors has =) T diagonalizable. 4. V= Span Zv, Iv, -- S

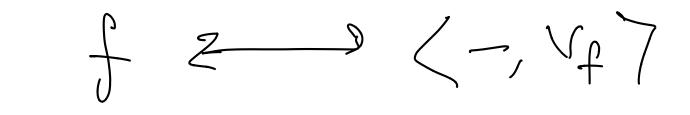
V= Span ZTU, --. J

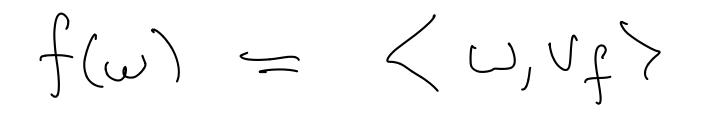


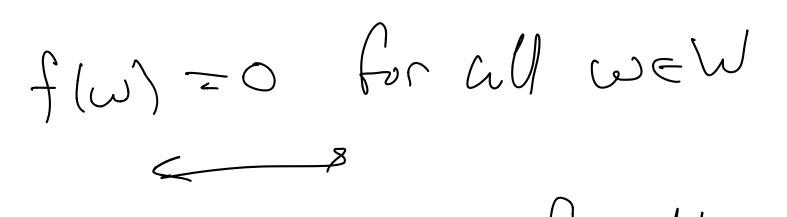
Inner Products and Ducl Space f.d. Vinner prod Space over IR. There and canonical iso. $f: V \longrightarrow V^*$ \sim (-) < -, $\sqrt{7}$ Recall WCV

 $W = \{f \in V^*, f(w) = 0\}$ frad we w

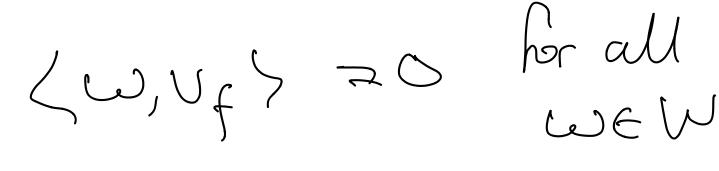






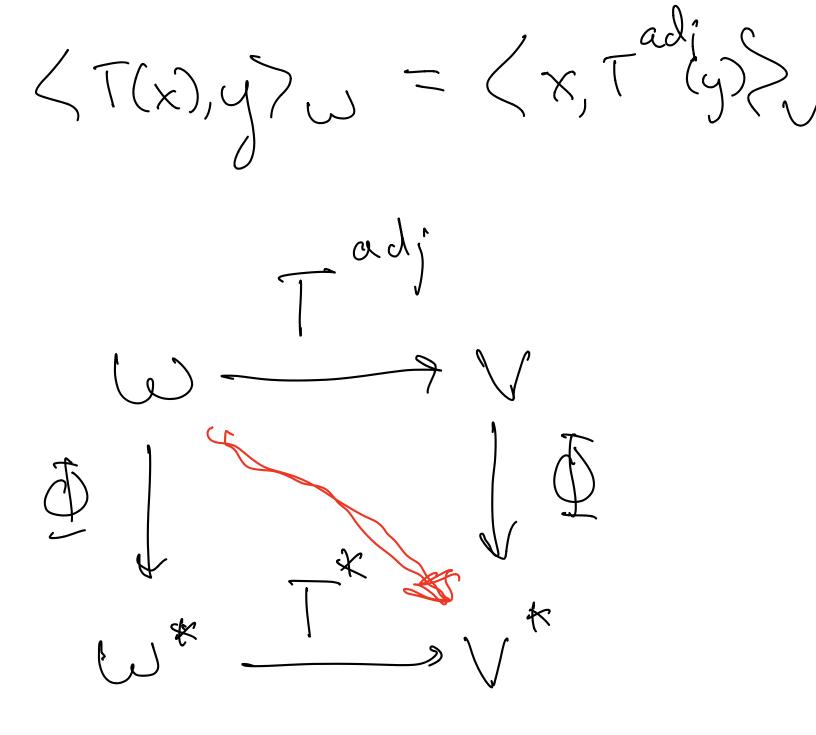






LE ULEW. 50 Wt K > WO undes the 130.

How does deal map and adjoint relate? $T: V \longrightarrow W$ (fdTPS)Tadj: W-JV



Thm. This duagram commutes i.e. \$\Delta To D

Proof. Pick wew. $(T^* \overline{\Phi})(\omega) = T^* (\overline{\Phi}(\omega))$ $\Phi(\omega) = f_{\omega} \qquad f_{\omega}(-) = \langle -\mu \rangle$ $T^*(f_{\omega}) = f_{\omega} \bullet T$ what does this do to a vector J^Z $(f_{\cup} \circ T)(v) = f_{\cup}(T(v))$ $= \left(\langle T(v), w \rangle \right)$

 $(\Phi \circ T \circ \dot{\Phi})(\omega \zeta$ $= \overline{\Phi}(\overline{(\omega)})$ $= \langle -, T^{\alpha \partial j}(\omega) \rangle \langle \langle \rangle \rangle$ what doe, this do to avector u? $\langle \cup, (a) \rangle \rangle$ Red boxes ore cqual by def.

 $= (T^* \Phi)(\omega) = (\Phi T^{ad})(\omega)$ as functionals, ble agree on arbitrary vev. $= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \frac{1}$ Can Solve her Say, Tadj: $T^{adj} = \overline{P}^{-1} \overline{T}^* \overline{D}$ Explicity, if Pison D.A. basis of V gison o.n. basis of W

B* dual bases well recover the fact that Tradil P = (Tradil P)[]t]pt None of this work, over C. Complex mer product spaces

cre werrd!