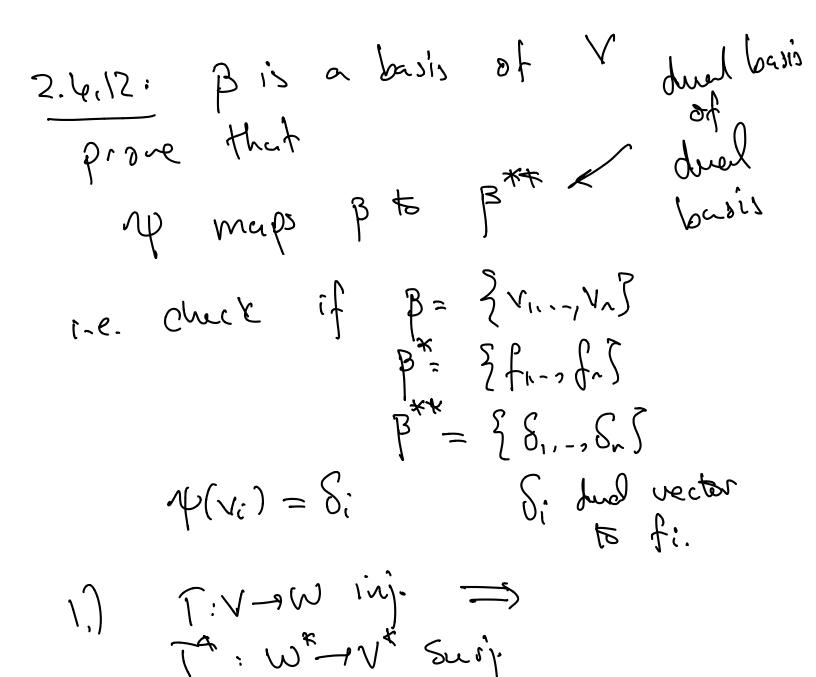
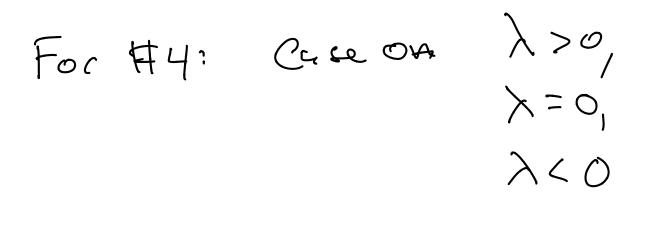
HW 1:



J



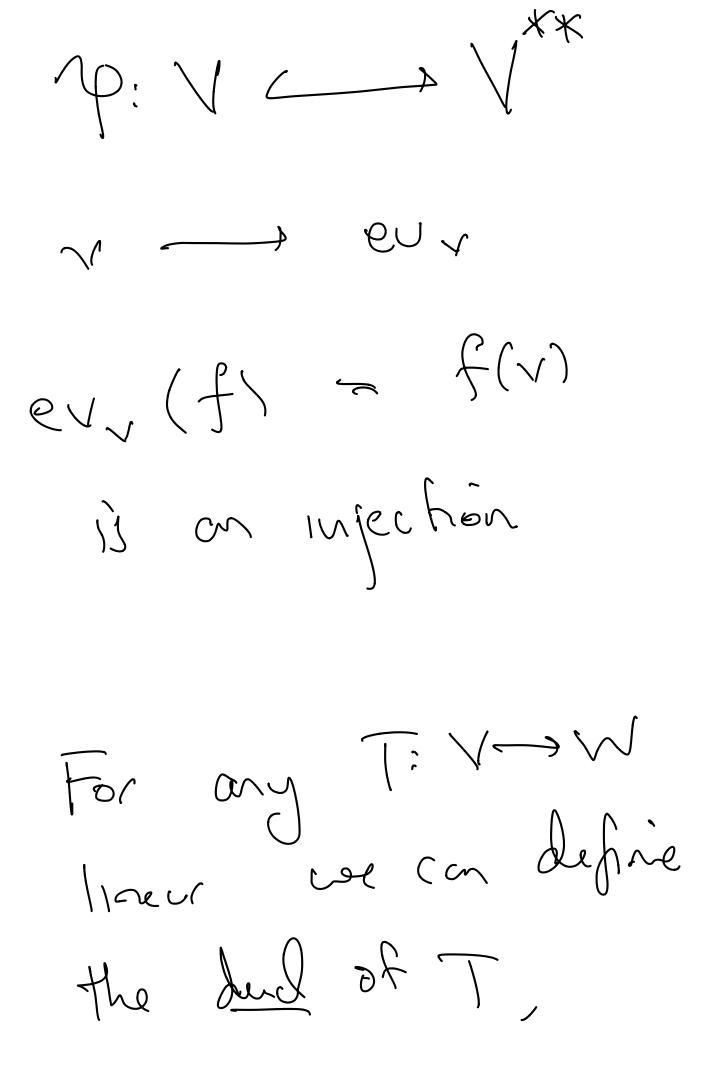
More on dual Space Fruector spere, \mathbf{N} Qn

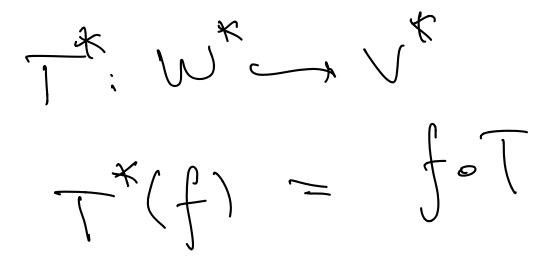
Onal Space $\sqrt{K} = f(V,F).$ V = f.d. with basis $\beta = \frac{2}{3} V_{11} - \frac{1}{3} V_{-1}$ Jud basis V & Z fi--, fr/ $f_i : \bigvee - F$ $f_i(v_j) = \begin{cases} i = j \\ i \neq j \end{cases}$ VSV, but this requires picking a busid.

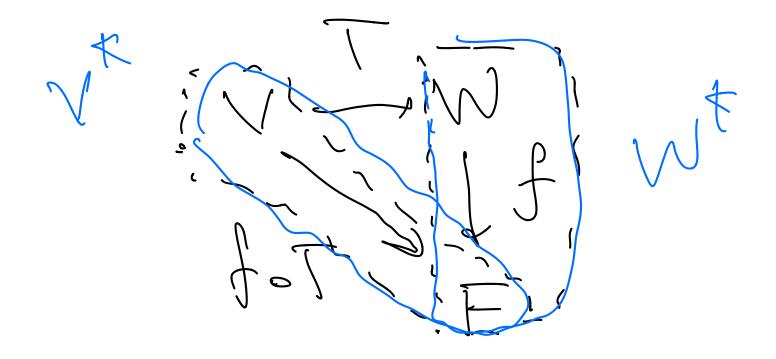
Ronk: if V f.J. Inner grod Space, Var V is Canonical $\vee \longrightarrow \langle -, \vee \rangle$ + 7 V is infinite dimensionel, V¥√* $l' = \frac{1}{2} (a_i)_{i=1} = \frac{1}{2} |a_n| < \omega$ aiemag Ex: $l^{\infty} = \begin{cases} (\alpha_i)_{i=1} \\ \alpha_i \in \mathbb{R} \end{cases}$: Sup $|\alpha_i| < n \\ i \neq n \end{cases}$

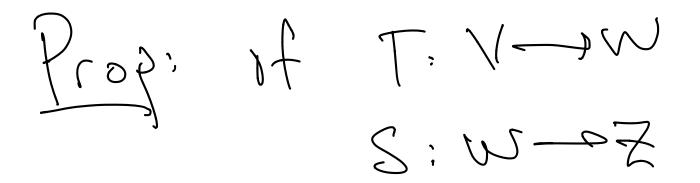
turns out $(l')^* = l^{\infty}$ l but these ore not iso. as IR wector spaces.

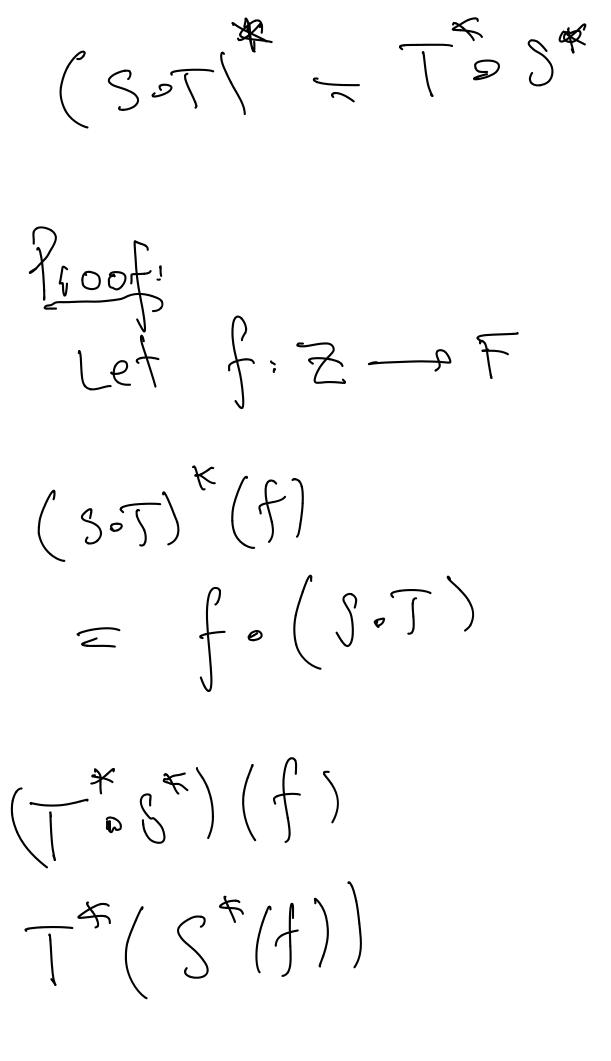
Proden: "dual basis" is no longer a spanning set? Hann, threis Cnnonral mapinto VX+;









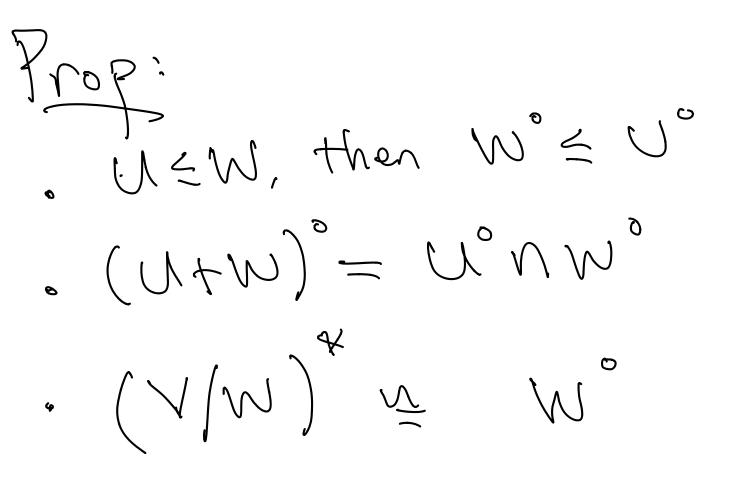


 $= T^{\star}(f \circ S)$ $= \left(f_{o} \right) a \left(f_{o} \right)$ ble composition is association, we have fo (SoTI = (foS) ot, so we're done. When V,W Cre f.d. UN bases B, V

k , t

 $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{y}$ Def: WCY Subspace the annhibitor of W $W^{\circ} = \{f \in \mathcal{N}^{\mathsf{E}} : f(w) = 0 \\ \text{for all we w} \}$ all functionals that Kill W. W & V*

W is a generalization of an orthogonal Conflement to an arbitrary vector space.



Tt' Suppose feW. Then f(w)=0 Tropf: Since UEW, for all we N. ueU. f(u) = 0 for all So feu.

Suppose fe Nonwo then f(u) = 0 for all usu f(w) = 0 for all we w Jina MAM = Zuaw: wew

f(u+u) = f(u) + f(w)=0*0=0.

 $f \in (U + w)^{\circ}$

If $f \in (\Pi + M)_{o}$ $f(x) = 0 \quad \text{for all } x \in UrW$ $f(x) = 0 \quad \text{for sond}$ $x = UtW \quad \text{for sond}$ for sond Uw

UC UtW

WCUtw

 $\mathcal{U} = \mathcal{U} \mathcal{V} \mathcal{O}$

w= Orw

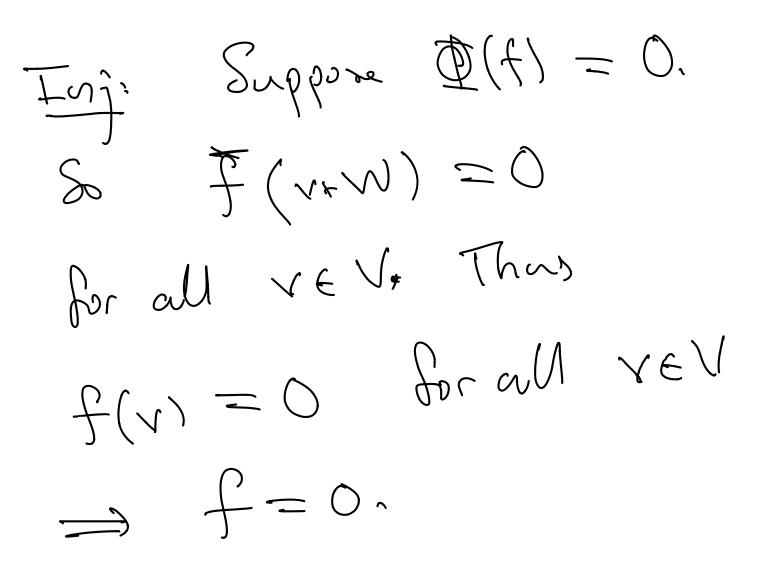
So f Kills

v and w

 $\Rightarrow fevinv.$ Défine a map Φ ; $W^{\circ} \rightarrow (V/W)^{*}$ Vary $\Phi(f) = \tilde{f}$ where $f(\mathcal{V}).$ f(1+M) =note that f is

vel definet because $If \chi + W = \chi + W,$ $H_{m} = V + W$ for Some WEW, ond $= f(\chi)$ F(V+W) $-f(\sqrt{+}\omega)$ blc flw)=0 as few = f(v') + f(w)= f(1)ß $= \overline{f}(\gamma' + W)$

Claim: I is on 130. Can easily check that t is linear.



Surj:

$$Pick f \in (V/W)^{k}$$
.
 $Wont t = construct g$
 $with$
 $\Phi(q) = f$.

Define g(x) = f(x+w). Define g(x) = f(x+w). w = 0 + w $b | c w \in w$ $b | c w \in w$ $g(\omega) = f(\omega, w) = f(0, w) w$ (note that g is also linear). S_{2} $q \in \mathcal{W}^{2}$

$$\Phi(q) = \overline{g}$$

$$\overline{g}(v + w) = g(v) = f(v + w)$$

$$S_{5} \quad \overline{g} = f$$

$$\Phi(q) = f \quad as \quad desired. \quad \square$$

Cor: dim M° = dim MN = din V - din W.

Ronk: this Shows that if Vir Inner prod. Space Worg wit ble same dim.

Didn't get to, but insightful: Prop: V, W f.d. T: $V \rightarrow W$ $Ker(T^*) = Im(T)^{\circ}$ Proof. QEKRI(Tr) $T^{*}(q) = 0 \implies q \circ T = 0.$

 $(g \circ T)(v) = 0 \quad \text{for all}$ $\vee \rightleftharpoons g(T(\gamma)) = 0$ fos all 11. $() g \in Im(T)^{\circ}$

If T: V-,V then rank nullity Says

din kerter din Inte den V den VerTrden ImT=den V Since dun V= dun V* and dim (Im T)° $= dim \sqrt{Im(T)}$ = din Kar (T)) din Ker T* = din Ker T din ImT*- dim ImJ rank T*= rank [. So

This prove the fact that "row vank" = "(Junson rank" from 331An

