Countability: A set X is called Countable if there's a bijection X - 7 4 for YCNI $\mathbb{Z},\mathbb{Q},\mathbb{Z}[x]$ EX: IR is not countable Any Finite set is Countrible

Prop: Frank products of Countable Sets

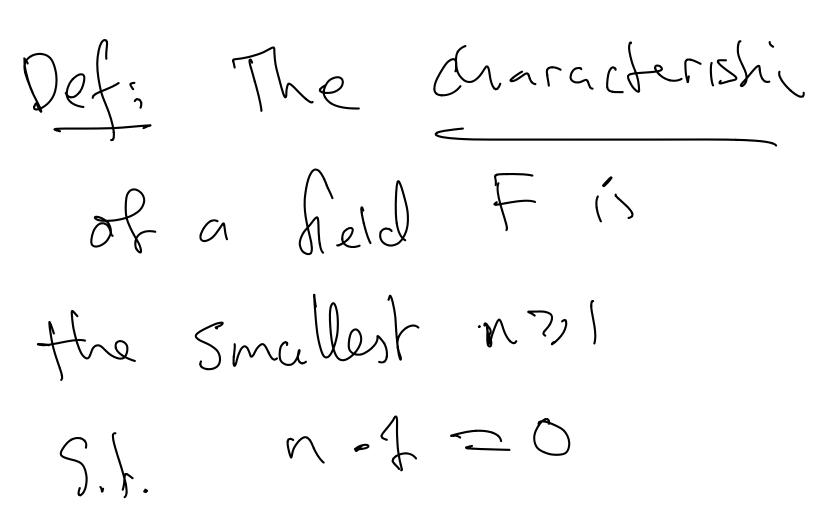
are countable.

Prop: Constable arions of countrial sets are Counterble. i.e. of Xi ore Countable, 30 is () () () () () ()

Hint For problem 3: First Show Met IR is infinite dimensional

over q. Then try and we second prop. Hint for proden 4: Use Zorn's lemma to Construct maximal Qui und. Subort of Spanning Set and Show it's a bads.

Show that Want ho $n \cdot 1 \neq 0$ ether ltirmit for -Mannes M $b - f \neq Q$ PR Some Prime P. \int IFp = 30,1,--, p-)? Ex. $f \cdot f = 0$

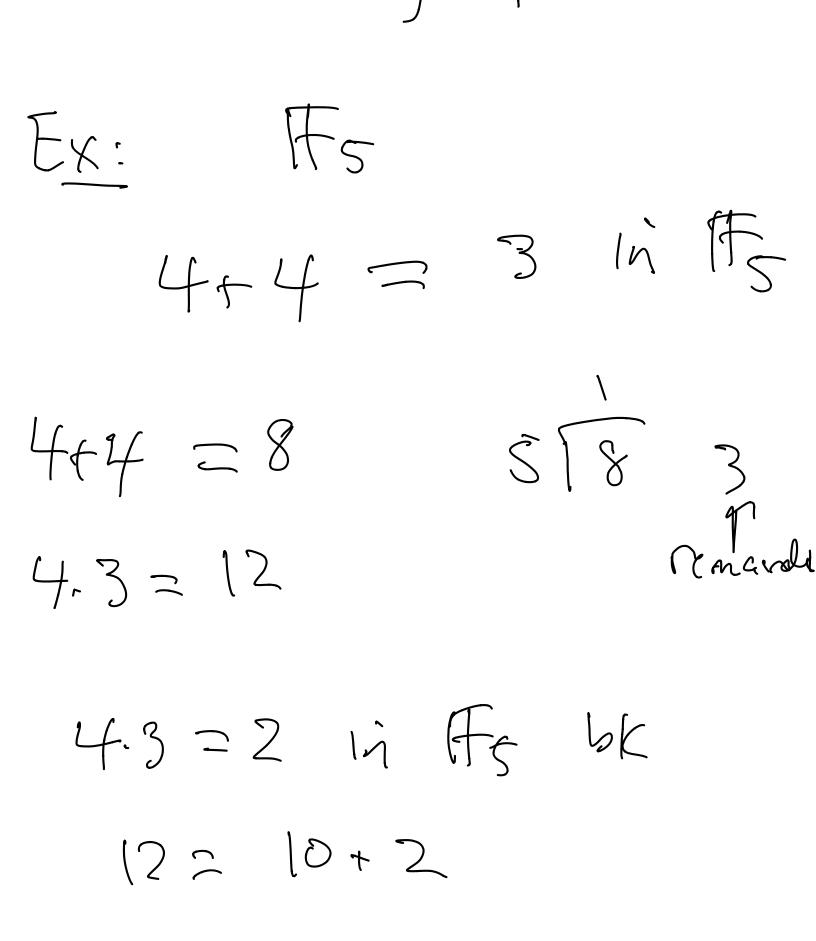


If No such h exist,

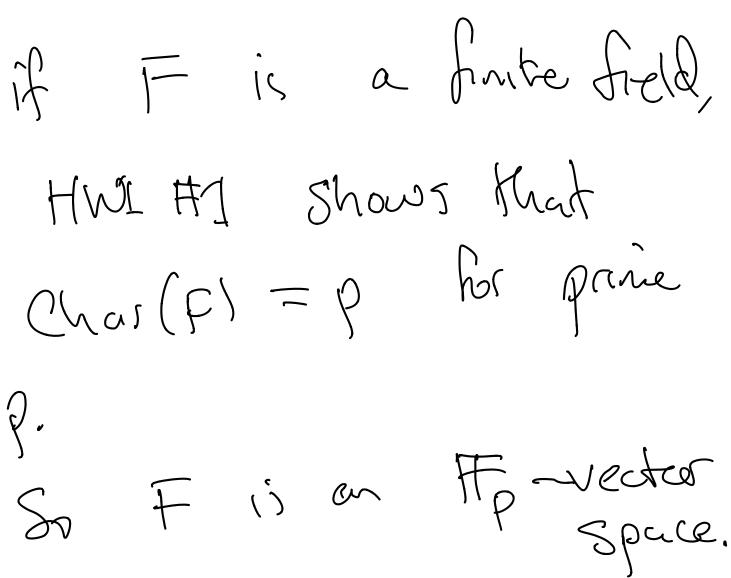
Char(F) = 0.

Hint for 1: Moch wath Chara chernfic. (i.e. use smallest Such n). Use the fact that Interpers are

products d'égrine. Finite fields $F_{p} = 20, 1, 2, ..., p-15$ P Prime addition/waltiplication ore "mod p": r.e. take remainders after disision by P.

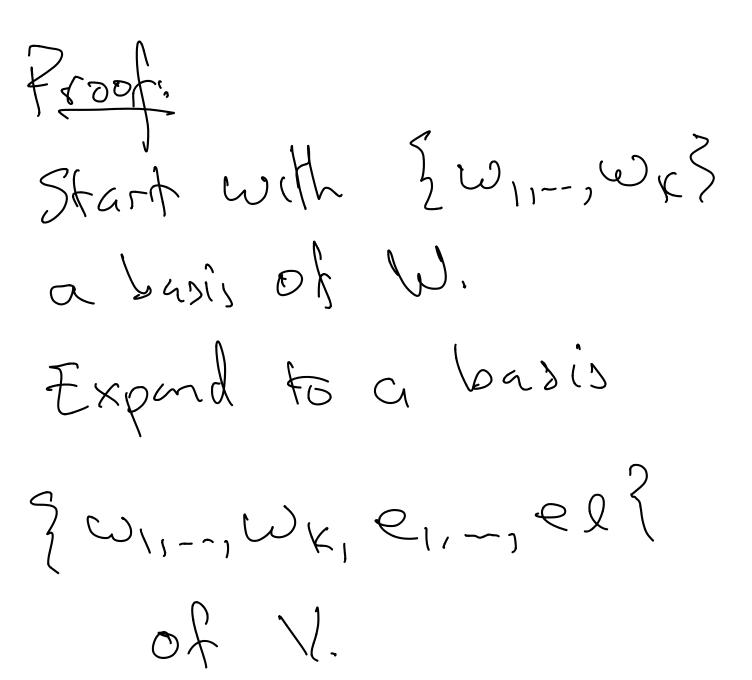


Ex: |+|=0x+08/20



Fis Ant, its Since Frite dimensional ES FS Fp V/W Z/W: VEV? Z V+W: VEV? $(\gamma + W) + (\gamma + W) = (\gamma + V) + W$ $C \cdot (V + W) = (C \cdot v) + W$

Prop: dim (V/W) = dim (V) - dim (W)



Claim: Ze, tW, m, elt WS is a basis for V/W.





CI(CI+W) + · · · · + CQ(CQ+W)

 \geq $\delta + W$

 $(C_1e_1+\dots+C_{QQ})+W = O+W$



Cleit--- tCeel - w has Sout WEW.

 $w = d_1 w_1 + \dots + d_k w_k$

 $C_{Q}, t \dots + C_{Q}C_{Q} = d_{Y}\omega_{Y} = \dots = d_{Y}\omega_{Y} = C_{Q}\omega_{Y}$ = a A C; d = 0ble Zeinel, winnwrz a 62813.

To see it spans V/W, Pick V+W C 1/W VZ CRIF-1 + CREFQW, F--FQWK V+W $C(e_1, \dots, e(e_1, e_2, e_3, \dots, e_3, \dots$ $C(e_1+w)_{1.1}+C(e_1+w)$

+ g/ (w/tw/ tra + gx (wktw) OFW blc wiew $= C_1(e_1+W) + \cdots + C_2(e_1+W)$ So vere done.



Define T: V - P V/W $\int (\gamma) \geq \gamma + \mathcal{W},$ Tis alinece fransformetion and T is Surjective dim(V) - dim kerT t din in T Zin Kert Jin Kert Jin V/W

 $Ver(T) \equiv W$ dim V = dim W/W

 $Mer(T) = \frac{2}{3}x:T(x) = 0$ in \sqrt{m}

QTN

V+W = O+W

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