HW 6 #2:

V real inner product space:
$$A = [T]$$

$$M_A^C = M_A^{R}$$

MA splits into district linear factor
in CIXJ blc A is dragonalizable
over C.
$$m_{H}^{c} = (x \cdot a_{1}) - (x \cdot a_{k})$$

 $m_{A}^{c} = m_{A}^{R}$
 \longrightarrow complex roots come in
complex pairs



JJJ Look at my Solutions for full details! pair up factor up conjugate to get real quadrahic factor. 100% at 7.2.7 7.2.6: Examples of Jorden Forms: More $E_X: A =$ $C_{A} = (x^{2})^{2}(x^{2})^{2}$ Sum of block sizes his $\lambda = 2,3$ is 2. Faither have 2x2 block or has 1x1 Glocks.

$$\begin{array}{cccc} A-2L & & \begin{pmatrix} 9 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ \end{pmatrix}$$

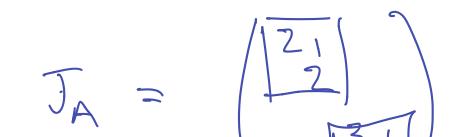
$$dim(E_2) = dim(ker(A-ZI)) = 1$$

=) 1 block for $\lambda = 2$

$$\begin{array}{c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{array}$$

din
$$(E_3) = din (ker(A-3I)) = 7$$

=) 1 block Cor $\lambda = 3$



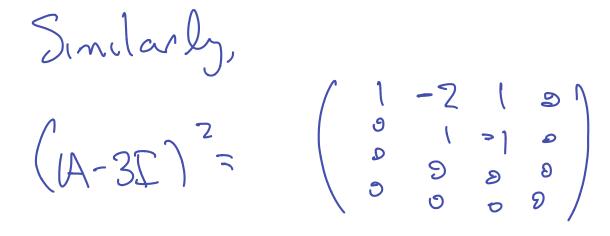
3 How to get a basis that gives JR?

Cycles ho Construct disjoint each block.

 $(A-2I)^2 = 0$ $(A-2I) \neq 0$ $\left\{ (A-2I)v, v \right\}$

 $(A-2I)^{2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{pmatrix}$

 $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ Ker $((A-2E)^2)$ = Span $\begin{cases} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{cases}$ C = 0- 2 C F d = 0Take $V = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $(A-2I) = \begin{pmatrix} & & \\ & & \\ & & \\ & & \\ & & \end{pmatrix}$



 $\begin{cases}
 a - 2b + c = 6 \\
 b - c = 6
 \end{cases}$ $\alpha - b = 0$ = a=b=c

 $V = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}^{2} = Span \begin{cases} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^{2}$ $V = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$$(IA-3I)_{X} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{cases} 2 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

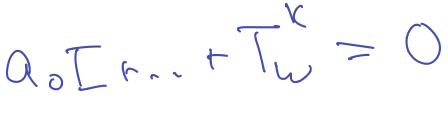
$$B = \begin{cases} 2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

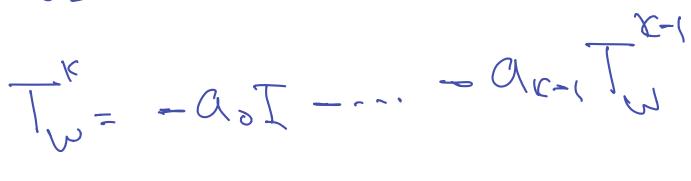
Rational Canonical Form VEV T:Y-rV K-dim. W= Span ZV, TV, --, TK-1 72

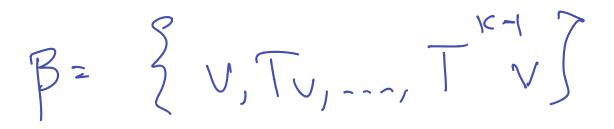
 $C_{T_W} = Q_0 + Q_1 X_{F^{---}} + X_{K}$

C-H Says that









 $\begin{bmatrix} T \\ T \\ W \end{bmatrix}_{3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 \\ 0$

Companión matrix hor 1 the polynomial Gor-n+ QueiX + X. I taked more about companion matrices week 4 RCF: We can find a basis Jof V S.t. $\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$ Cr is companion matrix her Some poly. 9, (x)

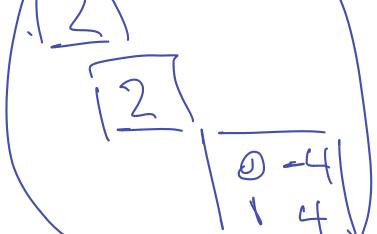
g. 1gz/... / gr.

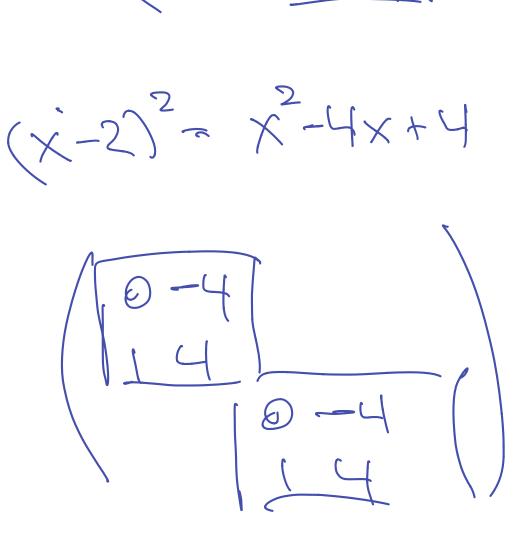
Turns out: RCF will always exist!!

The golynamids gigz, --, gr are called the Invariant factors of

How to compute RCF? • My is the largest invariant factor · Product of Invariant fuetors = CT Ex: A E My (IR) $C_{A} = (X-1)^{4}$ $M_{A} = (X-1)^{2}$

that are possible RCF ? List Invariat Inclors: $\chi - 2, \chi - 2, (\chi - 2)^{2}$ 02 $(X-2)^{c}(X-2)^{c}$





How to compute Invariant Factors:

For Small matrices, Sometimes just hoscer based off of Constraints and what CT B.

Algorithm: XI-A Use vou/column operations to two XI-A into a dragonal matrix that looks like $\left(\begin{array}{c}1\\3\\g^{2}\\g^{2}\\g^{2}\end{array}\right)$ g, [gel--- | gr Then gui->gr are the Invariant factors of A.

$$E_{X}: A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$
$$\times I - A = \begin{pmatrix} X - 1 & -1 & -1 \\ -1 & X - 1 & -1 \\ -1 & -1 & X - 1 \end{pmatrix}$$

$$\begin{pmatrix} X-1 & -1 & -1 \\ -(& X-1 & -1 \\ -1 & -1 & X-1 \end{pmatrix} \xrightarrow{P_2 \to P_1} \begin{pmatrix} 1 & (-X & 1) \\ -Y_2 \to P_2 \\ T \end{pmatrix} \begin{pmatrix} X-1 & -1 & -1 \\ -1 & -1 & X-1 \end{pmatrix}$$

$$\frac{(1-x)k_{1}+k_{2}-k_{2}}{k_{1}+k_{2}-k_{2}}\begin{pmatrix} 1 & 1-x & 1 \\ 0 & x^{2}-2x & -x \\ 0 & -x & x \end{pmatrix} -C_{1}+C_{2}-C_{2}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & x^2 - 2x & -x \\ 0 & -x & x \end{pmatrix} \xrightarrow{P_2 r R_3 - 1 R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & x^2 - 2x & -x \\ 0 & x^2 - 3x & 0 \end{pmatrix}$$

$$(x-2)(3+C_2-1)(2) \qquad (1 \circ 3) \qquad (0 \circ -x) \qquad ($$

$$\frac{-C_{3}+C_{3}}{T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \chi & 0 \\ 0 & 0 & \chi^{2}-3\chi \end{pmatrix}$$

$$\frac{1}{T} = \frac{1}{T} \int_{0}^{0} \int_{0}^{1} \int_{0}^{1}$$

RCF: Elementing during form.

The RCF above is the usual RCF. There's a different form Using demention durison instal

of Invariant factors that your book + lecture do.

Here's how it works:

G= finner fix as of irred. polynomiels. a product

Thm: There is a basis & S.t. $[T]_{\mathcal{F}} = (C_{\mathcal{C}})$ Where $C_i = Companion metrico of$ p. Pi P. P.fili for some i. The polynomeds appearing in this form

are called the elementing duriner of T. How to find? Use these facts: 1. Product of elementary divisors = C_T 2. lan of elementary divisors = MT 3. For each fi irred. factor of Gr # elementary durines corresponding to fi = dim Efc di $E_{f_i} = Ker(f_i(T))$ $d_i = dag(d_i).$

EX: J. R-PIR³ T(x) = Ax $A = \begin{pmatrix} 0 & 2 & 0 & -4 & 2 \\ 1 & -2 & 0 & -4 & 2 \\ 1 & 0 & 1 & -3 & 2 \\ 1 & -2 & 1 & -3 & 2 \\ 1 & -4 & 3 & -3 & 4 \end{pmatrix}$ $(x_{r2})^{2}(x-2) = f_{1}^{2}f_{2}$ $C_{0} =$ Can only have Single elementary divisor corresponding to X-Z. 2 X-2(Compute that f, (A) = $\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -12 & 0 \\
0 & 0 & 0 & -12 & 0 \\
0 & 0 & 0 & -12 & 0 \\
0 & 0 & 0 & -12 & 0
\end{pmatrix}$

dim $E_f = 2f$

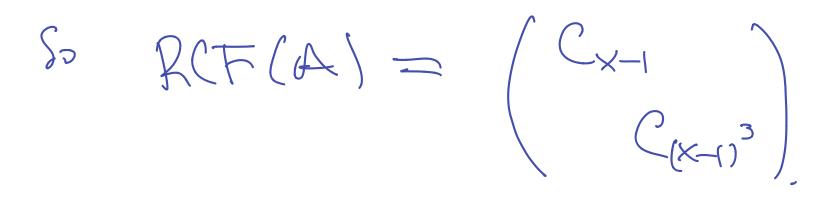
elementury dimensions for $f_1 = 4f_2 = 2$.

Must be

 $\{ \chi_{+2}^{2}, \chi_{+2}^{2} \}$

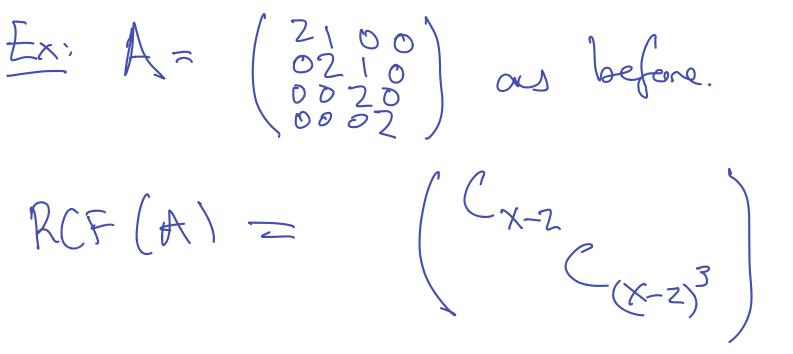
S RCF(T) $Z \begin{pmatrix} C_{x+2} \\ C_{x+2} \\ C_{x-2} \end{pmatrix}$

 E_{X} : T: $\mathbb{R}^{4} - \mathbb{R}^{4}$ T(x) = Ax $A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ $C_{T} = (x-2)^{4} = f_{1}^{4}$ Can check that $M_T = (X-2)^3$ and that dim Ef. = 2. So there are 2/1 = 2 elementing divisors corresponding to f. The constraints force then to be $\{X-1, (X-1)\}^3$?



How. to find a basis that gives RCF? Very Simlar to SCF.

Companion matrix « Cyclec Subspace generated by a vector.



V, E Ker (T-ZI). Then $T(v_1) = 2v_1$, so $W_1 = Spon \{v_1\}$ is T-cyclic with $\left[\mathcal{T}_{w} \right] = \left[2 \right] = C_{x-2},$

 V_{2} with $(T-2I)^{3} = 0$ but $(T-2I)^{2} \neq 0$

Then

 $W_{2} = Spon \left\{ V, TV, T^{2}V \right\}$ T-cyclic with $\left[T | W_{2}\right] = \begin{pmatrix} \vartheta & \vartheta & -4 \\ J & \vartheta & 4 \end{pmatrix} = C_{(X-Z)^{3}}$

So [V1, N2, AV2, AV2] is a basis that games RCF. Make Sure to pick N, & Spon Zvz, Auz, Avz, J.

Explacity, may Choose $V_{1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Ex: $A = \begin{pmatrix} 0 & 2 & 0 & -4 & z \\ 1 & -3 & 0 & 0 & z \\ 1 & 0 & 1 & -3 & z \\ 1 & -2 & 1 & -7 & 2 \\ 1 & 4 & 3 & -3 & 4 \end{pmatrix}$ Le Saw that RCF(A)Saw that RCF(A) $\begin{pmatrix} C \chi^{2} \chi^{2}$ <u>___</u>

So basis will be of the form $\{V_{1}, AV_{1}, V_{2}, AV_{2}, V_{3}\}$ N, E Ker(A+2I)V2 E Ker (ArZI) and N2 & Span ZN, ANS V3E Ker (A-ZI)

 $A_{V_{l}} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad Av_2 = \begin{pmatrix} 2 \\ -2 \\ 0 \\ -2 \\ 4 \end{pmatrix}$ A-2I = $\sqrt{3}$ $\approx \left(\begin{array}{c} 1\\ 1\\ 1\end{array}\right)$

is a basis that give

RCF.

Remark: You can easily get the list of elementary dursurs from invariant factors and vice -versa. Suppose CT = fi --- fix Invasiont factors

g.,--, g. $g_1 = f_1 - \cdots f_k$ gz= f. ... f. azy $g_{r} = f_{1} - f_{k}$ elementary dursors for f_{z} one given by J_{z} (ajc $\neq 0$) $\begin{cases} f_{i} & \dots & f_{r} \end{cases}$ Simley if guen elem. divisors Efinitie for each fi

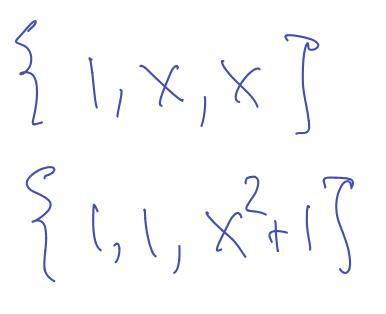
· order each list so that powers are non-decreasing. o Add 1's to beginning of list until all lusts have Some # of clevents. take product of fin for all i. $E_{X:} C_{t} = (x+i)^{2}(x-2)^{2}$ Invariant Factors:

 $\int (x_{H}) (x-2), (x-1) (x-2) (x-2)$ $\geq 2g_{1}g_{2}J$ $q' = (x+1)_{(x-5)_{x}}$ g2 ~ (×+1)'(×~2) Elen. divisors $= \frac{2}{2} \times \frac{2}{(X-2)^{2}} \times \frac{2}{(X-2)^{2}} \times \frac{1}{(X+1)^{2}}$

 $E_{X:} \quad C_{+} = \chi^2 (\chi^2_{+}) (\chi^2_{+})$

Elen duisors,

5x, x, x+1, x+2, x+2, x+2



 $\{X_{t}Z, X_{t}Z, X_{t}Z\}$

Invariant factors:

 $\{(X+2), X(X+2), X(X+1)(X+2)\}$