Quadrahi Forms Q: V-IF with Q(x)=B(x,x) for some symmetric biliner B Qui called quadrahi form Note: Q(ax) = a2Q(x) so Q is not linear. Q[x] = B(x,x) Z Cy \_\_\_\_\_ B(x,y) = 2 [B(xey)-B(2)-B(y])

rs a bypechion. Can Still talk about matrix of Q, etc. Over 12, theory of queelratic forms is nice. LQJp is Symmetric => deagonalizeble by Spectral Hum. if p= ¿un-jun brois of V  $O_1 \wedge \cdot$ 

and y= {w,-, wn } eigenbadi tet IXJy= (CIJCn). Then in y- coordinates, we concentre  $Q(C_{1}, C_{n}) = \lambda_{1}C_{n}^{2} + \cdots + \lambda_{n}C_{n}^{2}$ where hi is the eigenvalue har wo'.  $\frac{E_{x}}{E_{x}} = \frac{q(x,y)}{q(x,y)} = \frac{x^{2}-2xy}{y^{2}} + \frac{y^{2}}{y^{2}}$   $\frac{1-1}{p} = \begin{pmatrix} 1-1\\ -1 \end{pmatrix} \qquad p = \frac{1}{2}e_{1}e_{2}f$  $E_0 = Span \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$  $E_2 = Span S([])$  $\gamma = \begin{cases} \left( \frac{1}{\sqrt{2}} \right) \\ \left( \frac{1}{\sqrt{2}} \right) \\ \left( \frac{1}{\sqrt{2}} \right) \\ \left( \frac{1}{\sqrt{2}} \right) \end{cases}$  $if [x]_{\gamma} = (C_{1}, C_{2})$  $Q(c_1,c_2) = 2c_2^2$ 

If B, B2 : VXV -> E cre heo bilineur horms, Bi and Bz are called equivalent if thre is T: V-1V 150. 5.1 B(Try, Tw) - B2(u, v) for all u, well. (Such a Tri called on isometry) in matrix land, this corresponds to a change of basis:  $B_1 \sim B_2 \iff [B_1]_{\beta} = C^{2}[B_2]_{\beta}C$ fir some invertible matrix C. Study biliners (and quedrahi) Forms up to equivalence. For real quadratic forms, nice result,

$$\frac{E_{X:}}{Q_{1}(x,y)} = x^{2} - 2xy + y^{2}$$

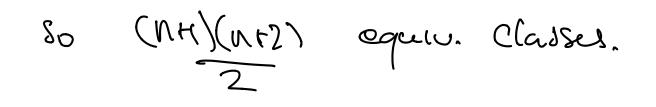
$$Q_{2}(x,y) = 2x^{2} + 4xy + 3y^{2}$$
we have with  $p = \frac{1}{2} - \frac{1}$ 

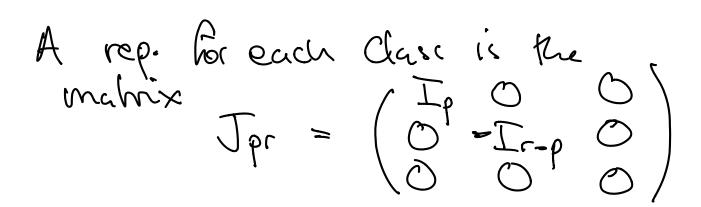
$$\left[ \left( \begin{array}{c} 2 \\ 2 \end{array} \right) \right]_{3} = \left( \begin{array}{c} 2 \\ 2 \end{array} \right)$$

One can cheet from their that rank  $Q_1 = 1$  index  $Q_1 = 1$ vanie  $Q_2 = 2$  index  $Q_2 = 2$ So  $Q_1 + Q_2$ .

How many different equilable  
Classes are there?  
dim 
$$V = u$$
  
possible rank:  $0,1,-,v$   
For rank = k, index can be  $0,1,-,k$ .  
So kFC choices for  
each choice of rank.  
 $\sum_{i=1}^{n} r_{i} = (n_{Fi})(n_{F2})$  choices

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Where p = rankr = index

First example of a cononcoal form, Will Falk about Canonical forms for lines operators later.