The Galois Correspondence



3.) $L \cong L' \rightleftharpoons H$ and H' are Conjugate in G. In particular, J Gal (K/L) J' Gal (K) O(L)) = for DEG. 4.) LIF is Galais (HEG Induced by (gal(L|F) 2- G|H the map $\sigma \rightarrow \sigma l_{L}$

Examples

1.) Last week we saw that Op(52,63/10 has Galois gp Z/2Z × Z/ZZ and we computed the fixed field of all subgps:

Q(F, 5) 213 51,03 Q (53) 31,73 Q(52) {1,07] Q(56) 1, 5, 7, 5, 1P 0: 52-7-52 \sim , $\sqrt{z} \rightarrow \sqrt{z}$ -3-2-53 53-953

Our computation last week Says this the full list of Subfields.









<6 auto. _ all chars are ante.



Gal(K(Q) = {1,0,02,2,02,02,02 $G_{al}(k|\Phi) \cong S_{3}$ Subgroup lattice of Sz: $S_3 = \langle (123), (121) \rangle$



us he Galois theory gues field lattice



3 Gabie ext¹ 1 quadrahie ext¹

Need to compute fixed fields of the Subgroups. Alternate way from last week: $3\sqrt{2}$ $3\sqrt{2}$ $3\sqrt{2}$ 3 [2 3 \ 2 *. .* (23)(23)ć : fixes 3 [2 →] K <~? Q(32) C Krithas degree 3 from

deagroom. Q(352) has Congutation. degree 3 via $\implies K \stackrel{\langle c \rangle}{=} \mathbb{Q} (3 \overline{c}).$ Similarly, K < (13)7 Z $\mathcal{P}(3 \mathbb{Z} \mathbb{Z}_3)$ X < (1217 - $\mathbb{P}\left(\begin{array}{c}3\\2\\3\end{array}\right)$

Finall, we see $Q(\hat{s}_3)$ is fixed by σ Sr $Q(\hat{s}_3) \subset K$

and has degree 2 $= \chi \quad \langle \nabla 7 = Q(\hat{S}_3).$ Ex: Last time coe Saw @(x, i52)/@ X= {143 has P8. Galois 97 B= 11-F3





K = (facisz)ζ 2 2 ζ Z quachic ext?

3 91

quadrati ext



Degree 4 Qxtersions; $(b|c \times 4 - 2x - 2)$ is icred $Q(\alpha \setminus$

 $Q(\beta)$ ($b/c x^2 - 2x^2 - 2$ (rred.)

 $Q(x) \neq Q(\beta)$ ble Bis not real.

 $Q(\overline{13}, \overline{152})$

has degree 4.

ble ist & R Q(TL) CIR.

How to figure out where to place the fields in the deagram?

just check which field is Fixed by what,

From table: i(2 fixed)by $26, 76^3 = 3$



-- equal-ty ble of

degrée reasons.



 $i(c = i\delta_2 \cdot \delta_3 = i\delta_2 \cdot (z^2 - 1)$ Can check Smilarly which Subgps the quarters correspond to.Remains to Find fixed

fuld of last hus

Subgps.



\bigcirc	- 2	ß	B
\backslash	2	3	cf

(trom fabe) ~0 -(14)(23)(C]= ((3)(24)

2-3 ~o(<> p) = X+P 263 (arp) =

Q(x-p) Q(x+p)

degree 4. hare

Galois orbit of X-B, X+B:

X-B, 2rB, -2+B, -2-B

=) from prop. last week that minimal poly. of 20B, 2+B have deg 4.

PG-BI Q(KrB) deyree ٢ $(\dot{\varphi}(x,i(z), \cdot))$ (P(B) (F3,156) (Jag) (Jag (p(~ Q(1(3) Giscis G(iE)