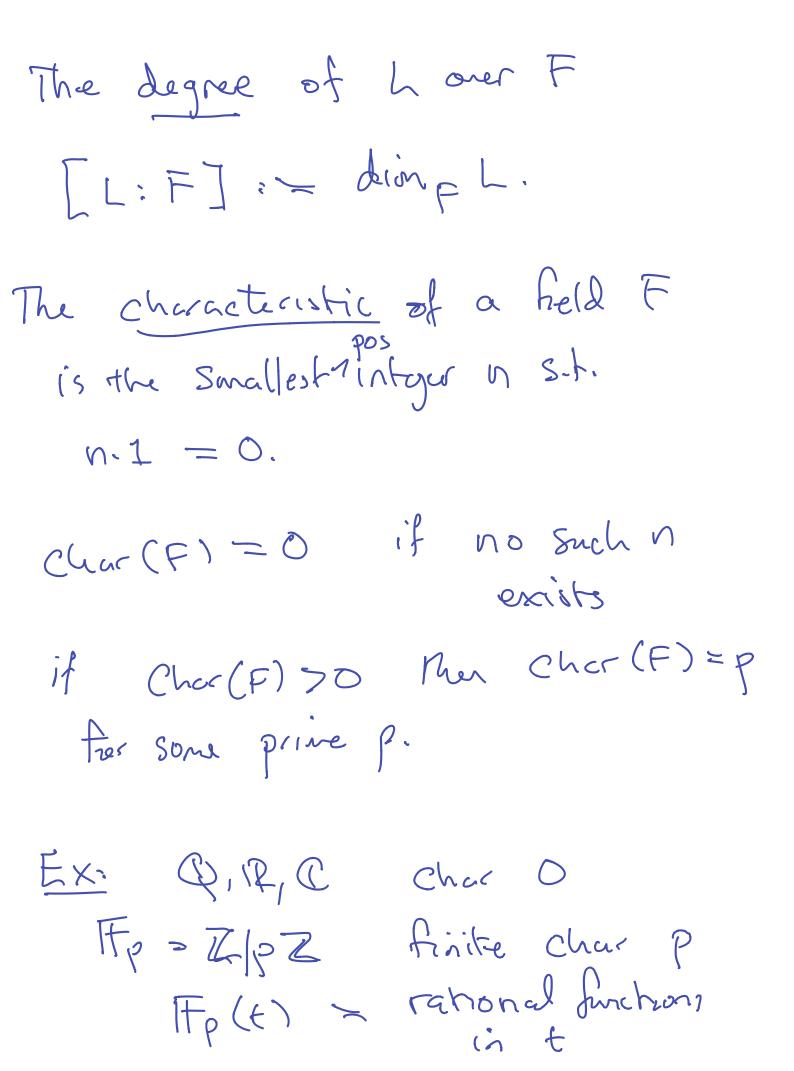
Contact.

Eval: tsmits@math.ucla.eda OH: Tues. 11:30-12:20 Th: 5:30-6:30

Website: can get to via CCCE.

Field Extensions is a fueld extension L/F I F are both helds. IF FCL (F subfield of C) L is an F-vector Main point. Space via multiplication.



infinite field of char p.

M/F XEL. d'il algebraic if plas20 for some ge FEX].

L/F is algebraic if all elts are algebraic.

Otherwise, extension is transcendental.

a algebraic Lecture Recap: J! monic (rred. ma ∈ FZXT) wa(a) = ∂ called minimal poly. of d.

· ma(x) | p(x) her any p(x) EF(x) $\omega | p(\alpha) = 0.$ $F(\alpha) = F - span of \alpha$. if del, L/F, then F(a) is Smallest Subpeld of L contraining F and L. $F(\alpha):F7 = deg ma = n.$ Basis is given by \$ 1,0,0²,--, 0⁻¹ }

Exi -
$$C = |R(i)$$

 $m_i = \chi^2 + i \in REX$
 C has basis $\{1,i\}$ as an $|R + i.s.$
 $Q(52)$ $m_{52} = \chi^2 - 2 \in Q[x]$
 $EQ(52): Q[= 2$
basis $\{2,i\}^2$?
if we view $f2 \in IR$
 $m_{52} = \chi - 5Z \in RE[x]$
 $i.o. R(52) = R$.
 $For any noo, \chi^2 - 2 \in Q[x]$
is Einestein at 2 \Rightarrow inted.

[Q(S2):Q] = Nbasis { 1, 12, 14. --]

How to compute minand polynomial? · Algebraic manipulations Une lineco algebra. θ

XEL/F Ta: 1-92 $\times \rightarrow \not \land \times$

F-liner map

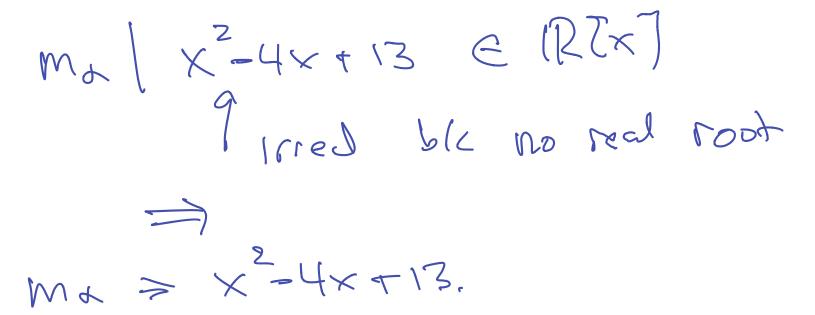
Turns out that (in the lincor Ma = MTar algebra Sense)

In pacticular, MTal CTa So we can quickly get Candidate minimal polynomials.

Ex: C/IR $\alpha = 2+3i$ What is ma?

 $\chi - 2 = 3i$ ($\chi - 2$) = -9

 $x^{2} = 4x + 13 = 0$



Alt: Let's conquite Ta. D/IR has basis Ziriz

 $T_{d}(i) = \alpha = 2i3i$ $T_{d}(i) = \alpha = -3i2i$

 $\begin{bmatrix} 7x \end{bmatrix} = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$

 $C_{T_{\alpha}} = \chi^2 - 4\chi + 13.$ Same as before Shows Cra is irred w/ a as a root =) equals Md.

In particular, this computation Shows that

[18(a):17] = 2

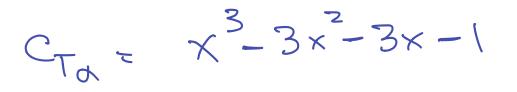
 $2\left(\begin{array}{c}1\\1\\1R(2)\\1\cdot 2\\1\end{array}\right)$ B(v) = C1.e. as expected.

 E_{x} : $\alpha = 1 + 3 [z + 3] [4]$ What is the degree of Qlar/Q? Ma?

VIEW $d \in \mathbb{Q}(3(z)).$ (352)Q(352)(Q has degree 3 blc $\mathcal{O}(\mathcal{F})$ X-2 EQ[x] lj Icred, ()

Q(352)/Q has busin {1,362, 364} aver Q. What is Tx? $T_{\alpha}(i) = \alpha = 1i^{3} 52 i^{3} 54$ $T_{\alpha}(i) = 2 i^{3} 52 i^{3} 54$ $T_{\alpha}(i) = 2i i^{3} 52 i^{3} 54$ $T_{\alpha}(i) = 2i i^{2} 5i^{3} 54$

 $\begin{bmatrix} T_{\alpha} \end{bmatrix} = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$



Note Cta has an Eisenstein translate at 1, 1.e. Ga(XH) is Eisenstein

So [Qual: QT = 3 blc found irred. poly of degree 3 that Kalle a (i.e. CTa = Ma !)

and in purticular, $Q(\lambda) = Q(35)$ Not dorions how to Show equality without a degree argument. Examples I didn't got to in discussion. tours of fields K [L:F] = [L:K][K:F]F

 $m_{G_2} = \chi^2 - 2 \in \mathbb{Q}[\chi]$ E_{X} , $Q(c_{z})$ $M_{GFZ} = X^{C} - Z \in O[X]$ $G\left(\begin{array}{c}13\\P(G)\\12\end{array}\right)$ $\Rightarrow \left[Q(\sqrt[6]{z}) : Q(\sqrt[5]{z}) \right] = 3.$ (1,)Note 652 is a root of $\chi^3 - \sqrt{2} \in Q(S_2) [x]$ => equals min. poly of 6 (2 over Q(52) (bk has same degree as min. poly!) Not immediately clear that x3-FZ @ Q(JZ)[x] 13 creel.

This strows how field theory cm Show polynomials are irred.

Ex: α root of $\chi^{3}-6x^{2}+9x+3 \in Q[x]$ Eisenstein at 3

$(\beta(\alpha))$	76 M	Ta 62 8	204)
13 Q		Q(v2):(
7	ad 2	43.	
Shaving	12 isn'	f in the	e Span
	31,0,0		
hard			

chat in Ex: a = [3+252 LQ(a):Q7?d2 = 3+252 => ~ 13 root of $\chi^2 - (3rZ(2)) \in \mathbb{Q}(JZ)ZXT$ $\Phi(\alpha)$ 50 Q(1)/Q 1 42 has degree ()(z)2004. 12 (ext has degle 1 (lop $Q(J) = Q(J_2), i.e. \Delta \in Q(J_2).$

Wart to Solve $d = a + b \int 2$ $a, b \in \mathbb{Q}$ $\alpha^2 = \alpha^2 + 2b^2 + 2ab/2$ $\Rightarrow \begin{cases} 3 = a^2 + 2b^2 \\ 1 = ab \end{cases}$ take a=6=1. Note that $(1+52)^2 = 3-252$ is indeed true, so actually ae Q(d) = Q(dz)have $E\varphi(z): \varphi \neq z$ 22 d Schoffer $x^4 - 6x^2 + 1 = 0$ Note:

by manpulation. Weat cec

did Shows X⁴-6x²+1 is <u>reducible</u> in QTXJ, Not at all obvious!