Homework 7 for Math 131AH Honors Analysis

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Fall 2016, UCLA

Due on Tuesday, November 22.

Rudin, p. 98 (ch. 4): 1, 3, 18.

(1) Let X and Y be metric spaces. Show that a function $f\colon X \to Y$ is continuous if and only if the restriction of f to every compact subset of X is continuous.

(2) Show that a nonempty metric space X is connected if and only if every continuous function $X \to \mathbf{Z}$ is constant.

(3) Let $f\colon [0,1] \to [0,1] \times [0,1]$ be a continuous surjective mapping. Show that f cannot also be injective.