Math 115AH Linear Algebra. Homework 5

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Due Friday, October 30.

Problems from Hoffman-Kunze:

Section 3.3: 3, 7.

Section 3.4: 2, 3, 4, 8, 10, 11, 12. (Hint: 8 is really an eigenvalue problem.)

(1) (an old exam problem) Suppose that $T\colon V \to W$ is linear, $S\colon W \to V$ is linear, and $\dim V > \dim W$. Let $1_W\colon W \to W$ be the identity on W: $1_W(w) = w$ for all $w \in W$.

(a) Can $TS = 1_W$? Proof or counterexample.

(b) Can $ST = 1_V$? Proof or counterexample.

(2) (another exam problem)

Suppose that $T\colon V \to V$ is linear, and that $T^3 = 0$.

(a) What can you say about the relation of $\ker(T)$ and $\mathrm{im}(T^2)$? Prove your answer.

(b) What can you say about the relation of $\ker(T^2)$ and $\mathrm{im}(T)$? Prove your answer.