

Math 115AH Linear Algebra. Homework 4

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Due Monday, October 26.

Problems from Hoffman-Kunze:

Section 3.1: 1, 3, 4, 7, 12, 13.

Section 3.2: 2, 3, 4, 6, 8, 9, 11.

Let X, Y, Z be sets, and let $g: X \rightarrow Y$ and $f: Y \rightarrow Z$ be functions. (We are just talking about sets and functions, not vector spaces and linear maps.) For the following statements, prove the correct ones and give counterexamples to the incorrect ones. (Appendix A2 discusses functions.)

- (1) If $f \circ g$ is surjective (also called “onto”), then g is surjective.
- (2) If g is injective (also called “one-to-one”), then so is $f \circ g$.
- (3) If f and g are bijective (injective and surjective), then $f \circ g$ is bijective.
- (4) If $f \circ g$ is injective, then g is injective.
- (5) If f is onto, then so is $f \circ g$.
- (6) If g is onto, then so is $f \circ g$.
- (7) If f and g are onto, then so is $f \circ g$.
- (8) If $f \circ g$ is onto, then f is onto.
- (9) If $f(A \cap B) = f(A) \cap f(B)$ for all subsets A and B of Y , then f is injective.
- (10) Give an example of a set S , a proper subset $T \subsetneq S$, and a bijection $f: S \rightarrow T$. (Hint: S cannot be finite.)