Solutions to the Exercises of Section 2.1.

2.1.1. **Proof.** Let $\mathcal{C}$ be a complete class of decision rules, and let A denote the class of admissible rules. We are to show $A \subset \mathcal{C}$. If $\delta \notin \mathcal{C}$, then there is a $\delta_0 \in \mathcal{C}$ that is better than δ. This shows that δ is not admissible, i. e., $\delta \notin A$. So, $\mathcal{C}^c \subset A^c$, or $A \subset \mathcal{C}$. (A^c denotes the complement of the set A.)

2.1.2. **Proof.** Let $\mathcal{C}$ be essentially complete and let δ be admissible. If $\delta \notin \mathcal{C}$, then, using the essential completeness of $\mathcal{C}$, there exists a $\delta_0 \in \mathcal{C}$ such that δ_0 is as good as δ. But since δ is admissible, δ_0 cannot be better than δ. Thus δ_0 and δ have the same risk function and are equivalent.

2.1.3. If C is a complete class, then for every rule δ not in C, there exists a rule δ' in C such that δ' is better than δ. But if δ' is better than δ, then δ' is also as good as δ, so C satisfies the definition of being an essentially complete class.

2.1.4. Let C be a minimal complete class. We may break C up into equivalence classes, $C = \cup_{\alpha \in A} C_\alpha$, where for each α, any two elements of C_α have the same risk function, and if $\alpha \neq \beta$ then $C_\alpha \cap C_\beta = \emptyset$ and elements of C_α and C_β have different risk functions. Then by the axiom of choice, there is a set D consisting of one point from each of the sets C_α for $\alpha \in A$. Then the set D is minimal essentially complete.

2.1.5. The proof uses the result of Exercise 2.1.3, that every complete class is essentially complete. Suppose that C is a complete class and that C contains no proper subclass that is essentially complete. Then it can contain no proper subclass that is complete either, so C is minimal complete. Moreover, C is essentially complete, and so is minimal essentially complete.

2.1.6. **Proof.** Let A denote the class of admissible rules and suppose that A is complete. We must show that no proper subclass of A is complete. Let A' be a proper subclass of A so that $A - A'$ is nonempty. Let $\delta \in A - A'$. If A' were complete, then there would be a rule $\delta_0 \in A'$ that is better than δ. But then δ would not be admissible and so would not be in A. This contradiction completes the proof.

2.1.7. Suppose that Θ consists of one point, say $\Theta = \{0\}$, and that the risk set is the open interval, $S = (0, 1)$. (Such a situation occurs if $\mathcal{A} = (0, 1)$ and $L(0, a) = a$ for all $a \in \mathcal{A}$.) If C_1 denotes the class of decision rules with rational risk points, and if C_2 denotes the complement of C_1, then both C_1 and C_2 are complete, but $C_1 \cap C_2$ is empty and so is not essentially complete.