Solutions to the Exercises of Section 1.6.

1.6.1. If δ_0 is minimax, then by definition, $\sup_{\theta\in\Theta} R(\theta,\delta_0) = \inf_{\delta\in D^*}\sup_{\theta\in\Theta} R(\theta,\delta)$. This implies that $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)$ for all $\delta\in D^*$. This in turn implies that $R(\theta',\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)$ for all $\theta'\in\Theta$ and for all $\delta\in D^*$.

Conversely, if $R(\theta',\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)$ for all $\theta'\in\Theta$ and for all $\delta\in D^*$, then $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)$ for all $\delta\in D^*$. This in turn implies that $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \inf_{\delta\in D^*}\sup_{\theta\in\Theta} R(\theta,\delta)$. But the left side cannot be less than the right side; so we have equality, showing that δ_0 is minimax.

1.6.2. If δ_0 is ϵ-minimax for some $\epsilon>0$, then by definition, $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \inf_{\delta\in D^*}\sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$. This implies that $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$ for all $\delta\in D^*$. This in turn implies that $R(\theta',\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$ for all $\theta'\in\Theta$ and for all $\delta\in D^*$.

Conversely, if for some $\epsilon>0$, $R(\theta',\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$ for all $\theta'\in\Theta$ and for all $\delta\in D^*$, then $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$ for all $\delta\in D^*$. This in turn implies that $\sup_{\theta\in\Theta} R(\theta,\delta_0) \le \inf_{\delta\in D^*}\sup_{\theta\in\Theta} R(\theta,\delta)+\epsilon$, showing that δ_0 is ϵ-minimax.

1.6.3. If τ_0 is least favorable, then by definition, $\inf_{\delta\in D^*} r(\tau_0,\delta) = \sup_{\tau\in\Theta^*}\inf_{\delta\in D^*} r(\theta,\delta)$. This implies that $\inf_{\delta\in D^*} r(\tau_0,\delta) \ge \inf_{\delta\in D^*} r(\tau,\delta)$ for all $\tau\in\Theta^*$. This in turn implies that $r(\tau_0,\delta') \ge \inf_{\delta\in D^*} r(\tau,\delta)$ for all $\tau\in\Theta^*$ and for all $\delta'\in D^*$.

Conversely, if $r(\tau_0,\delta') \ge \inf_{\delta\in D^*} r(\tau,\delta)$ for all $\tau\in\Theta^*$ and for all $\delta'\in D^*$, then $\inf_{\delta\in D^*} r(\tau_0,\delta) \ge \inf_{\delta\in D^*} r(\tau,\delta)$ for all $\tau\in\Theta^*$. This in turn implies that $\inf_{\delta\in D^*} r(\tau_0,\delta) \ge \sup_{\tau\in\Theta^*}\inf_{\delta\in D^*} r(\theta,\delta)$. But the left side cannot be greater than the right side; so we have equality, showing that τ_0 is least favorable.