Mathematics 170B – HW9 – Due Tuesday, May 29, 2012.

Problems 1,2,3 on page 326.

Definition. A positive integer valued random variable N is a stopping time with respect to the discrete time stochastic process $\{X_1, X_2, \dots\}$ if for every n, the event $\{N = n\}$ is determined by the values of $\{X_1, X_2, \dots, X_n\}$.

J_1. Suppose $\{X_1, X_2, \dots\}$ is a Bernoulli process. Show that $N =$ the time at which the second success occurs is a stopping time.

J_2. Show that if N is a stopping time, then so is $N_k = \min\{N, k\}$.

Wald's second identity states the following: If $X_1, X_2, \dots$ are i.i.d. and satisfy $EX_i = 0, EX_i^2 = \sigma^2 < \infty$, and N is a stopping time with $EN < \infty$, then $ES_N^2 = \sigma^2 EN$. You need not prove this; those of you in Math 171 may have done the proof.

J_3. Consider the simple symmetric random walk $S_n = X_1 + \cdots + X_n$, where $X_1, X_2, \dots$ are i.i.d. random variables taking the values ± 1 with probability $\frac{1}{2}$ each. For positive integers l, m, let

$$N = \min\{n \geq 1 : S_n = -l \text{ or } m\}$$

and $N_k = \min\{N, k\}$. (N, and hence N_k, is a stopping time, but you need not prove this – it should be obvious to you!)

(a) Use Wald's second identity to obtain an upper bound for EN_k that does not depend on k.

(b) Conclude that $EN < \infty$.

(c) Apply Wald's first identity to compute $P(S_N = m)$.

(d) Use Wald's second identity to compute EN.