Math 131BH, MS^2, Winter 01

Workshop #1

1. Let $f : \mathbb{Q} \to \mathbb{R}$ be defined by $f(x) = 0$ if $x < \sqrt{2}$ and $f(x) = 1$ if $x > \sqrt{2}$. Show that f is continuous on all of $\mathbb{Q}$.

2. Define functions $f_n : \mathbb{R} \to \mathbb{R}$ by

$$f_n(x) = \frac{nx}{1 + |nx|}.$$

Show that f_n is continuous for every $n \in \mathbb{N}$. For which $x \in \mathbb{R}$ is the function

$$f(x) = \lim_{n\to\infty} f_n(x)$$

defined and continuous?

3. Show that $f : \mathbb{R}_+ \to \mathbb{R}$ defined by $f(x) = \sqrt{x}$ is uniformly continuous, but the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$ is not.

4. For any metric space X, is the identity function on X continuos? Uniformly continuous? Prove your answers.

5. Let $[a, b]$ be an interval in $\mathbb{R}$ and f a continuous, real valued, one to one function on $[a, b]$. Prove that $f([a, b]) = [f(a), f(b)]$ or $[f(b), f(a)]$ (whichever one makes sense).

6. Let $f_1, f_2, ...$ be real valued continuous functions on a metric space X and assume for each $x \in X$, the sequence $(f_1(x), f_2(x), ...)$ is bounded. Define $f(x) = sup(f_1(x), f_2(x), ...)$ for each $x \in X$. Prove or find a counterexample: f is continuous.