Assignment 9 (Due Mar 14). Covers: Week 9 notes

- Q1 (a). Prove Lemma 1 from Week 9 notes. (Hint: In order to show that (a) implies (b), consider the supremum and infimum of X).

- Q1 (b). Use Lemma 1 to prove Corollary 2. (Hint: explain why the intersection of two bounded sets is automatically bounded, and why the intersection of two connected sets is automatically connected).

- Q2. Let I be an interval of the form $I = (a, b)$ or $I = [a, b)$ for some real numbers $a < b$. Let $I_1, \ldots, I_n$ be a partition of I. Prove that one of the intervals I_j in this partition is of the form $I_j = (c, b)$ or $I_j = [c, b)$ for some $a \leq c \leq b$. (This is used to prove Theorem 3. Hint: Prove by contradiction. First show that if I_j is *not* of the form (c, b) or $[c, b)$ for any $a \leq c \leq b$, then $\sup I_j$ is *strictly* less than b).

- Q3. Prove Lemma 4 from Week 9 notes.

- Q4. Prove Lemma 5 from Week 9 notes.

- Q5. Prove Lemma 6 from Week 9 notes. (Hint: Use Lemmas 4 and 5 to make f and g piecewise constant with respect to the *same* partition of I).

- Q6. Prove Proposition 7 from Week 9 notes. (Hint: First use Theorem 3 to show that both integrals are equal to $p.c. \int_{[\mathbf{P} \# \mathbf{P}']} f$).

- Q7. Prove Theorem 8 from Week 9 notes. (Hint: You can use earlier parts of the theorem to prove some of the later parts of the theorem. See also the hint to Q5).

- Q8. Prove Lemma 10 from Week 9 notes.

- Q9 (a). Prove Lemma 11 from Week 9 notes.

- Q9 (b). Prove Proposition 12 from Week 9 notes. (Hint: you will need Lemma 11, though it will only do half of the job).

- Q10*. Prove Theorem 13 from Week 9 notes. (Hint: use Q7, but avoid using Q8. For part (b): First do the case $c > 0$. Then do the case $c = -1$ and $c = 0$ separately. Using those cases, deduce the case of $c < 0$.)