Assignment 6 (Due Feb 21). Covers: Week 6 notes

You may use anything from Week 1-6 notes which does not introduce circularity. Some of the results here are also proven in the textbook (though the notation is a little different sometimes). You are encouraged to read the textbook; you are welcome to use the proofs from that textbook provided you phrase them in your own words.

- Q1(a) Prove Lemma 1 of Week 6 notes.
- Q1(b) Can you find two sequences $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ which are not the same sequence, but such that each is a subsequence of the other?
- Q2. Prove Proposition 2 of Week 6 notes. (Note: Proposition 2(b) has a very short proof).
- Q3. Prove Proposition 3 of Week 6 notes. (An advanced remark concerning Proposition 3(a): you may find yourself using (perhaps unknowingly) the "axiom of choice" when proving this Proposition. It is possible to avoid this axiom, though, by use of the well-ordering principle. If you do not know what the axiom of choice is, ignore this remark).
- Q4. Prove Lemma 5 of Week 6 notes.
- Q5. Prove Proposition 6 of Week 6 notes.
- Q6. Prove Proposition 9 of Week 6 notes.
- Q7. Prove Proposition 10 of Week 6 notes. (This can largely be done by applying the previous propositions and lemmas. Note that to prove (a),(b),(c) are equivalent, you do not have to prove all six equivalences, but you do have to prove enough at least three; for instance, showing that (a) implies (b), (b) implies (c), and (c) implies (a) will suffice (although this is not necessarily the shortest or simplest way to do this question)).
- Q8. Prove Proposition 12 of Week 6 notes. (Hint: you can use the fact that $\lim_{n\to\infty} a^{1/n} = 1$ (or Proposition 30 of Week 5 notes), combined with the squeeze test and the laws of exponentiation).

- Q9. Prove Proposition 13 of Week 6 notes. (Hint: from limit laws one can show that $\lim_{x\to 1} x^n = 1$ for all integers n. From this and the squeeze test one can show that $\lim_{x\to 1} x^p = 1$ for all real numbers p. From this and the laws of exponentiation one can prove Proposition 13.)
- Q10. Prove Proposition 15 of Week 6 notes.