Assignment 6 Due November 14 Covers: Sections 3.1-3.2; 4.1-4.4

- Q1. Do Question 1(abcdefgi) of Section 3.1 of the textbook.
- Q2. Do Question 1(acdefhi) of Section 3.2 of the textbook.
- Q3. Do Question 5(e) of Section 3.2 of the textbook.
- Q4. Do Question 6(ade) of Section 3.2 of the textbook.
- Q5. Do Question 1(abcdefgh) of Section 4.2 of the textbook.
- Q6. Do Question 25 of Section 4.2 of the textbook.
- Q7. Let U, V, W be finite-dimensional vector spaces, and let $S : V \to W$ and $T : U \to V$ be linear transformations.
- (a) Show that $\text{rank}(ST) \leq \text{rank}(S)$.
- (b) Show that $\text{rank}(ST) \leq \text{rank}(T)$.
- (c) Show that $\text{nullity}(ST) \geq \text{nullity}(T)$.
- (d) Give an example where $\text{nullity}(ST) > \text{nullity}(S)$.
- (e) Give an example where $\text{nullity}(ST) < \text{nullity}(S)$.
- Q8. Let A and B be $n \times n$ matrices. Prove (from the definition of transpose and matrix multiplication) that $(AB)^t = B^t A^t$.
- Q9. Let A be an invertible $n \times n$ matrix. Prove that $\det(A^{-1}) = 1/\det(A)$.
- Q10. Let A and B be invertible $n \times n$ matrices. Show that one there is a sequence of elementary row operations which transforms A to B. (Hint: first show that there is a sequence of row operations which transforms A to the identity matrix).