Assignment 4 Due October 31 Covers: Sections 2.3-2.4

- Q1. Do Exercise 5(cdefg) of Section 2.2 of the textbook.
- Q2. Do Exercise 1(aegij) of Section 2.3 of the textbook.
- Q3. Do Exercise 4(c) of Section 2.3 of the textbook.
- Q4. Do Exercise 10 of Section 2.3 at the textbook. (T_0 is the zero transformation, so that $T_0 v = 0$ for all $v \in V$.
- Q5. Do Exercise 1(bcdefhi) of Section 2.4 of the textbook.
- Q6. Do Exercise 2 of Section 2.4 of the textbook.
- Q7. Do Exercise 4 of Section 2.4 of the textbook.
- Q8*. Do Exercise 9 of Section 2.4 of the textbook.
- Q9. Let U, V, W be vector spaces.
- (a) Show that U is isomorphic to U.
- (b) Show that if U is isomorphic to V, then V is isomorphic to U.
- (c) Show that if U is isomorphic to V, and V is isomorphic to W, then U is isomorphic to W.
- (Incidentally, the above three properties (a)-(c) together mean that isomorphism is an *equivalence relation*).
- Q10. From our notes on Lagrange interpolation, we know that given any three numbers y_1, y_2, y_3, there exists an interpolating polynomial $f \in P_2(\mathbf{R})$ such that $f(0) = y_1$, $f(1) = y_2$, and $f(2) = y_3$. Define the map $T : \mathbf{R}^3 \to P_2(\mathbf{R})$ by setting $T(y_1, y_2, y_3) := f$. (Thus for instance $T(0, 1, 4) = x^2$). Let $\alpha := ((1, 0, 0), (0, 1, 0), (0, 0, 1))$ be the standard basis for $\mathbf{R}^3$, and let $\beta := (1, x, x^2)$ be the standard basis for $P_2(\mathbf{R})$.
- (a) Compute the matrix $[T]_\alpha^\beta$. (You may assume without proof that T is linear.)

- (b) Let $S : P_2(\mathbf{R}) \to \mathbf{R}^3$ be the map

$$Sf := (f(0), f(1), f(2)).$$

  Compute the matrix $[S]_\beta^\alpha$. (Again, you may assume without proof that S is linear).

- (c) Use matrix multiplication to verify the identities

$$[S]_\beta^\alpha [T]_\alpha^\beta = [T]_\alpha^\beta [S]_\beta^\alpha = I_3,$$

  where I_3 is the 3×3 identity matrix. Can you explain why these identities should be true?