Assignment 2 Due October 17 Covers: Sections 1.6-2.1

- Q1. Do Exercise 1(acdejk) of Section 1.6 in the textbook.
- Q2. Do Exercise 3(b) of Section 1.6 in the textbook.
- Q3. Do Exercise 7 of Section 2.1 in the textbook.
- Q4. Do Exercise 9 of Section 2.1 in the textbook.
- Q5. Find a polynomial $f(x)$ of degree at most three, such that $f(n) = 2^n$ for all $n = 0, 1, 2, 3$.
- Q6. Let V be a vector space, and let A, B be two subsets of V. Suppose that B spans V, and that span(A) contains B. Show that A spans V.
- Q7*. Let V be a vector space which is spanned by a finite set S of n elements. Show that V is finite dimensional, with dimension less than or equal to n. [Note: You cannot apply the Dimension Theorem directly, because we have not assumed that V is finite dimensional. To do that, we must first construct a finite basis for V; this can be done by modifying the proof of part (g) of the Dimension theorem, or the proof of Theorem 2.]
- Q8*. Show that $\mathcal{F}(\mathbf{R}, \mathbf{R})$, the space of functions from $\mathbf{R}$ to $\mathbf{R}$, is infinite-dimensional.
- Q9. Let V be a vector space of dimension 5, and let W be a subspace of V of dimension 3. Show that there exists a vector space U of dimension 4 such that $W \subset U \subset V$.
- Q10. Let $v_1 := (1, 0, 0)$, $v_2 := (0, 1, 0)$, $v_3 := (0, 0, 1)$, $v_4 := (1, 1, 0)$ be four vectors in $\mathbf{R}^3$, and let S denote the set $S := \{v_1, v_2, v_3, v_4\}$. The set S has 16 subsets, which are depicted on the reverse of this assignment. (This graph, incidentally, depicts (the shadow of) a *tesseract*, or 4-dimensional cube).
- Of these subsets, which ones span $\mathbf{R}^3$? which ones are linearly independent? Which ones are bases? (Feel free to color in the graph and turn it in with your assignment. You may find Corollary 1 in the Week 2 notes handy).