Assignment 1 Due October 10 Covers: Sections 1.1-1.6

- Q1. Let V be the space of real 3-tuples

  $$V = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbf{R}\}$$

  with the standard addition rule

  $$(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$$

  but with the non-standard scalar multiplication rule

  $$c(x_1, x_2, x_3) = (cx_1, x_2, x_3).$$

  (In other words, V is the same thing as the vector space $\mathbf{R}^3$, but with the scalar multiplication law changed so that the scalar only multiplies the first co-ordinate of the vector.)

  Show that V is *not* a vector space. (Which ones of the axioms I-VIII in the lecture notes (or (VS1)-(VS8) in the textbook) hold, and which ones fail?)

- Q2. (a) Find a subset of $\mathbf{R}^3$ which is closed under scalar multiplication, but is not closed under vector addition.

  (b) Find a subset of $\mathbf{R}^3$ which is closed under vector addition, but not under scalar multiplication.

- Q3. Find three distinct non-zero vectors u, v, w in $\mathbf{R}^3$ such that $\mathrm{span}(\{u, v\}) = \mathrm{span}(\{v, w\}) = \mathrm{span}(\{u, v, w\})$, but such that $\mathrm{span}(\{u, w\}) \neq \mathrm{span}(\{u, v, w\})$.

- Q4. Find a basis for $M^0_{2\times 2}(\mathbf{R})$, the vector space of 2×2 matrices with trace zero. Explain why the set you chose is indeed a basis.

- Q5. Do Exercise 1(abghk) of Section 1.2 in the textbook.

- Q6. Do Exercise 8(aef) of Section 1.2 in the textbook.

- Q7. Do Exercise 23 of Section 1.3 in the textbook.

- Q8*. Do Exercise 19 of Section 1.3 in the textbook. [**Hint**: Prove by contradiction. If $W_1 \not\subseteq W_2$, then there must be a vector w_1 which lies in W_1 but not in W_2. Similarly, if $W_2 \not\subseteq W_1$, then there must be a vector w_2 which lies in W_2 but not in W_1. Now suppose that both $W_1 \not\subseteq W_2$ and $W_2 \not\subseteq W_1$, and consider what one can say about the vector $w_1 + w_2$.]

- Q9. Do Exercise 4(a) of Section 1.4 in the textbook.

- Q10. Do Exercise 1(abdef) of Section 1.5 in the textbook.