Orbispaces uniformizations of sub-hyperbolic maps and their iterated monodromy groups

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Bonded orbit equivalence

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The orbit equivalence is continuous if there exists a continuous (i.e., locally constant) associated cocycle. It is bounded if the cocycle can be chosen to take a finite number of values (as a function of $x$) for every $g_1 \in G_1$. 
Example: torsion groups from the dihedral group

**Theorem**

Let $a, b$ be two homeomorphisms of the Cantor set $X$ such that $a^2 = b^2 = Id$. 

First examples of simple groups of subexponential growth were constructed using this method.
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Let $a, b$ be two homeomorphisms of the Cantor set $\mathcal{X}$ such that $a^2 = b^2 = \text{Id}$. Suppose that all orbits of the action of $\langle a, b \rangle$ are dense.
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We say that a correspondence $f, \iota : M_1 \rightarrow M$ is \textit{expanding} if $M$ is compact and there exists a metric on $M$ with respect to which $f$ is a local isometry and $\iota$ is locally contracting.
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Here a faithful action of \( G \) on \( X^\omega \) is self-similar if for every \( g \in G \) and \( x \in X \) there exist \( h \in G \) and \( y \in X \) such that \( g(xw) = yh(w) \) for all \( w \in X^\omega \).
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Example: p.c.f. rational functions

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If we remove all points of the post-critical set $P_f$, we get a correspondence $f, \iota : \hat{\mathbb{C}} \setminus f^{-1}(P_f) \rightarrow \hat{\mathbb{C}} \setminus P_f$ of trivial orbispaces.

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For example: the dihedral group is the iterated monodromy group of $z^2 - 2$; its limit dynamical system is conjugate to that of the Grigorchuk group.

All self-similar contracting groups with this limit dynamical system (conjugate to the tent map) have been classified and constitute a class of groups defined earlier by Z. Šunić.

All groups in this family are of intermediate growth, except for the dihedral group.
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Examples

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- Do there exist torsion “exotic” i.m.g.’s of rational functions with the Julia set equal to the sphere.
Questions

- Describe explicitly all iterated monodromy groups of orbispace uniformizations of a p.c.f. rational function.
- Interpret in group-theoretical terms the result of M. Bonk and D. Meyer on “unmating” rational functions.
- Are all “exotic” iterated monodromy groups of Lattès examples amenable? What is their growth?
- Is every “exotic” iterated monodromy group of $z^2 + i$ of intermediate growth?
- Do there exist torsion “exotic” i.m.g.’s of rational functions with the Julia set equal to the sphere. (They do for some polynomials with dendroid Julia sets, by a result of J. Cantu.)
Questions

- Study more general (not self-similar) groups boundedly orbit equivalent to iterated monodromy groups of rational functions.
Questions

- Study more general (not self-similar) groups boundedly orbit equivalent to iterated monodromy groups of rational functions. Some of them may come in self-similar families related to non-autonomous “rotated” matings, or other representations of the Julia set as continuous images of the circle or dendrites.