Math 170E

Lecture Notes Section 2.1 [*][†]

Discrete random variables

Instructor: Swee Hong Chan

NOTE: Materials that appear in the textbook but do not appear in the lecture notes might still be tested. Please send me an email if you find typos.

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[†]This notes is based on March Boedihardjo's and Jamie Haddock's notes from the past quarters, and I would like to thank them for their generosity. "*Nanos gigantum humeris insidentes* (I am but a dwarf standing on the shoulders of giants)".

1 Random variables

Simple definition: A **random variable** is just a random number, which we denote by X.

Textbook definition: Given a random experiment with outcome space S, a **random variable** is a function X that assigns one real number $X(s)$ to each s in S.

2 Random variables: Examples

Example 1. The outcome of a dice throw is a number from $\{1, 2, 3, 4, 5, 6\}$. This (random) number is a random variable X, and the probabilities are

- $P(X = 3) = 1/6$,
- $P(X \geq 5) = 2/6$,
- $P(2 \leq X \leq 4) = 3/6$.

Example 2. Consider the random experiment of flipping a coin (outcome is either head or tail). The random variables X that corresponds to coin flippping is

$$X(\text{head}) = 1; \qquad X(\text{tail}) = 0.$$

3 Discrete type random variables

A random variable X is of **discrete type** if the number of outcomes of X is *countable*.

For example, The random number X picked uniformly from the set $\{1, 2, 3\}$ is of discrete type.
The random number X picked uniformly from the interval $[0, 1]$ is NOT of discrete type.

Notation: Large-cap X is usually a random variable, while small-cap x is usually a deterministic real number.

4 Probability mass function

The **probability mass function** (pmf) f of a discrete random variable X is given by

$$f(x) := P(X = x),$$

for every real number x.

For example, the pmf for the dice random variable X is

$$f(1) = f(2) = f(3) = f(4) = f(5) = f(6) = \frac{1}{6};$$
$$f(x) = 0 \quad \text{for } x \notin \{1, 2, 3, 4, 5, 6\}.$$

5 Support of a random variable

The **support** of a random variable X is the set of points for which $f(x) > 0$.

In the dice random variable from Example 2, the support is the set $\{1, 2, 3, 4, 5, 6\}$.

For most discrete random variables, the support is equal to the outcome space. That is, every element in the outcome space has positive probability.

6 Cumulative distributive function

The **cumulative distributive function** (cdf) $F(x)$ of a discrete random variable X is the function

$$F(x) := P(X \le x),$$

for every real number x.

The cdf $F(x)$ for the coin random variable X is

$$F(x) = \begin{cases} 0 & \text{for } x < 0; \\ \frac{1}{2} & \text{for } 0 \le x < 1; \\ 1 & \text{for } x \ge 1. \end{cases}$$

7 Bar graphs and probability histograms

Two methods that we usually use to illustrate the pmf are **bar graphs** and **probability histograms**.

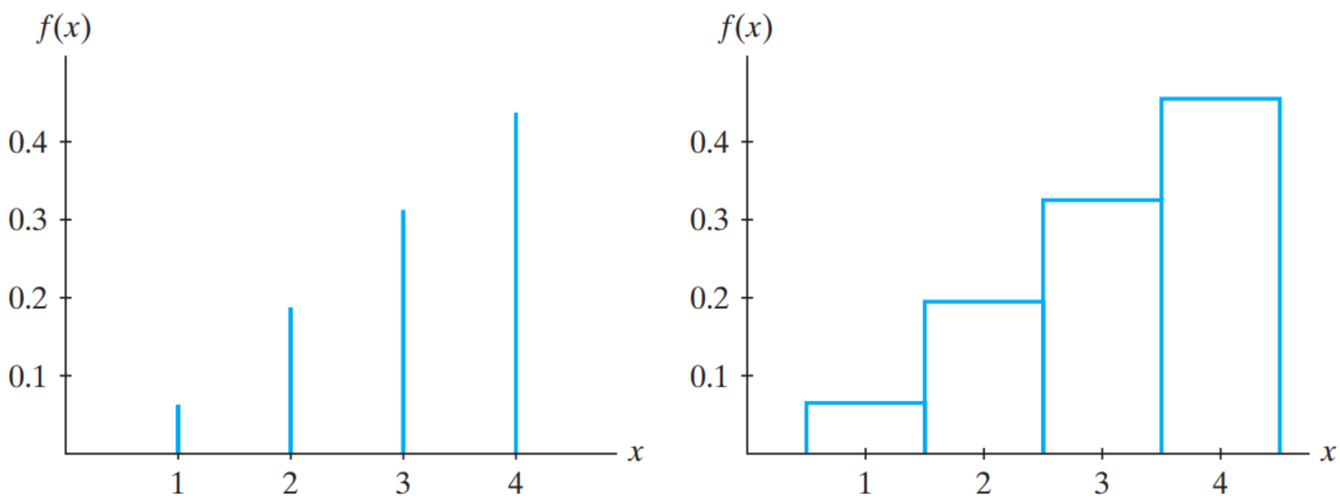

Figure 1: The **bargraph** and the **probability histogram** of the random variable X with pmf given below (taken from the textbook).

$$f(1) = 0.05; \quad f(2) = 0.2; \quad f(3) = 0.3; \quad f(4) = 0.45.$$

8 Uniform random variable

A random variable is **uniform** if the pmf is a constant function on the support. Equivalently, if every outcome in the support is equally likely.