Before starting 225C, you should be able to give detailed proofs (from first definitions) of the following. You should take 10 minutes per problem. You should also be able to check the proof yourself.

(1) Let $X = \cup_i A_i$ be a finite union of closed subsets A_i. Let $f_i\colon A_i \to Y$ be continuous functions, so that, $f_i|_{A_i \cap A_j} = f_j|_{A_i \cap A_j}$ for all i, j. Define $f\colon X \to Y$ by setting $f|_{A_i} = f_i$. Prove f is continuous.

(2) Let $\sim$ be an equivalence relation on a topological space and let $\pi\colon X \to X/\sim$ be the quotient map. Define the quotient topology on $X/\sim$ by stipulating $U \subset X/\sim$ is open if and only if $\pi^{-1}(U)$ is open in X. Prove that this is indeed a topology on $X/\sim$, and it is the largest topology which makes π continuous.

(3) Let X, Y be topological spaces. Define the product topology on $X \times Y$ by stipulating $W \subset X \times Y$ is open if and only if W is a union of sets of the form $U_\alpha \times V_\beta$ where U_α is open in X and V_β is open in Y. Prove that this is indeed a topology, and that it is the smallest topology which makes both the projection maps $X \times Y \to X$ and $X \times Y \to Y$ continuous.

(4) Let A be a deformation retract of X. Prove that A is homotopy equivalent to X.

(5) Prove that homotopy equivalence is an equivalence relation. That is, if X, Y, Z are topological spaces with X homotopy equivalent to Y and Y homotopy equivalent to Z, then X is homotopy equivalent to Z.

(6) Prove that the annulus $A = \{z \in \mathbb{C} \mid 1 \leq |z| \leq 2\}$ and the cylinder $S^1 \times [0,1]$ are homeomorphic, and either is homotopy equivalent to the punctured plane $\mathbb{C}^* = \{z \in \mathbb{C} \mid z \neq 0\}$, but not homeomorphic.

(7) Prove that any self-homeomorphism of the disk $D^n = \{x \in \mathbb{R}^n \mid |x| \leq 1\}$ restricts to a self-homeomorphism of the sphere $S^{n-1} = \partial D^n = \{x \in \mathbb{R}^n \mid |x| = 1\}$.

(8) Prove that D^{n+m} and $D^n \times D^m$ are homeomorphic.

(9) Prove that the pairs $(D^n \times [0,1], D^n \times \{0\})$ and $(D^n \times [0,1], D^n \times \{0\} \cup S^{n-1} \times [0,1])$ are homeomorphic.

(10) Let $\sim$ be the equivalence relation on D^2 which quotients $I = [0,1]$ to a point. Prove that the triples $(D^2, S^1 \cup I, 1)/\sim$ and $(D^2, S^1, 1)$ are homeomorphic.