HW 8

(Q-1) Show that for $M = S^2$, the Weingarten map $L = \pm\text{Id}$ by computing L_{ik} in a coordinate patch and raising an index.

(Q-2) Show that for $M = S^1 \times (0,1)$, L can be represented by the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

(Q-3) Find L for the torus from (HW-4, Q-1).

(Q-4) Prove that $II(X,Y) = \langle L(X), Y\rangle = \langle X, L(Y)\rangle$ for all $X, Y \in T_pM$.

(Q-5) Find the matrix (L^i_j) for a surface of revolution.

(Q-6) Let $f\colon V \to U$ be a surface of revolution. How are the $\overline{L}^\alpha_\beta$ related to the L^i_j?

(Q-7) Let $\gamma(s) = x(\gamma^1(s), \gamma^2(s))$ be a unit speed curve on a surface. Note (T, S, n) gives a positive orthonormal frame, with $n(s) = n(\gamma^1(s), \gamma^2(s))$ viewed as a function of s. Prove the following analogues of the Frenet-Serret equations:

$$\begin{aligned} T' &= II(T,T)n + \kappa_g S \\ N' &= -\kappa_g T + II(T,S)n \\ S' &= -II(T,T)T - II(T,S)S. \end{aligned}$$

(Q-8) Find the Gaussian and mean curvatures of the plane, sphere, and the torus.

(Q-9) Prove $H^2 \geq K$. When does equality hold?

(Q-10) Let $X, Y \in T_pM$ be orthogonal vectors. Prove

$$H = \tfrac{1}{2}\big(II(X,X) + II(Y,Y)\big).$$