HW 7

(Q-1) Prove that the meridian of a surface of revolution is a geodesic. Also determine which circles of latitude are geodesics.

(Q-2) Let M be a surface and Π a plane that intersects M in a curve γ. Show that γ is a geodesic if Π is a plane of symmetry of M, that is, the two sides are mirror images.

(Q-3) Let γ be a straight line on a surface. Prove that γ is a geodesic.

(Q-4) Suppose x is a coordinate patch with $g_{11} \equiv 1$ and $g_{12} \equiv 0$. Prove that the u^1-curves are geodesics.

(Q-5) If M is a surface of revolution and γ is a geodesic, prove that $r \cos \beta(s)$ is a constant where $\beta(s)$ is the angle between $\gamma'(s)$ and the circle of latitude (of radius r) through $\gamma(s)$.

(Q-6) Let $\gamma(t)$ be a geodesic not parametrized by arc length. Prove, for each $i = 1, 2$,

$$\frac{d^2\gamma^i}{dt^2} + \sum \Gamma^i_{jk} \frac{d\gamma^j}{dt} \frac{d\gamma^k}{dt} = -\frac{d\gamma^i}{dt} \frac{d^2 t}{ds^2} \left(\frac{ds}{dt}\right)^2.$$

(Q-7) Consider the patch $x(u, v) = (u, v, uv)$. Show that the non-unit speed curve $\gamma(t) = (t, -t, -t^2)$ is a geodesic, when parametrized by arc length.

(Q-8) Let $\alpha(s) = (f(s), g(s)$ be a simple unit speed plane curve. Let $x(s, t)$ be the surface $x(s, t) = (f(s), g(s), t)$. Let β be a fixed constant, and let $\gamma(\theta) = (f(\theta), g(\theta), \theta \tan \beta)$. Prove γ is a geodesic. (Note, θ is not the arc length.) Prove γ is a helix.

(Q-9) Find as many geodesics as you can on the surface $x^2 + y^2 - z^2 = 1$.

(Q-10) Find as many geodesics as you can on the patch $x(u, v) = (u, v, u^2 - v^2)$.